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MATHEMATICS IN M.C. ESCHER'S GALLERY

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Abstract: M.C. Escher, a Dutch graphic artist, is renowned for his masterful integration of mathematical concepts into his art. His work, particularly the piece *Gallery* exemplifies the seamless blend of art and mathematics, showcasing the beauty and complexity inherent in both fields. This paper delves into the mathematical structures underlying Escher's *Gallery* exploring the principles of complex transformations and wallpaper groups. By analyzing these elements, we aim to uncover the profound connections between art and mathematics, and how Escher's work challenges our perception of reality.
Keywords: M.C. Escher; Complex transformation; Wallpaper Group; Symmetry; Spiral; Infinity

1 INTRODUCTION

M.C. Escher, born in 1898, was a Dutch graphic artist whose work has left an indelible mark on both the art and mathematics communities, and has become a symbol of the intersection of these two seemingly separate fields. His unique ability to incorporate mathematical principles into his art has fascinated scholars and enthusiasts alike for decades. Escher's journey began with a passion for drawing and a keen interest in the natural world, which led him to explore the intricate patterns and shapes found in nature, such as the Fibonacci sequence, tessellations, and the golden ratio. Throughout his career, Escher created numerous works that showcased his mastery of the art of visual illusion and mathematical concepts. Among his most iconic works is *Gallery* (1957), a visual exploration of space, symmetry, and infinity. Through meticulous planning and execution, Escher created a self-referential artwork that blurs the lines between reality and illusion, inviting viewers to ponder the nature of perception and human experience [1].

Escher's spiral designs are another prominent feature of his work, demonstrating his deep understanding of mathematical concepts such as rotation and reflection. These designs often feature intricate patterns that seem to endlessly spiral outward or inward, captivating the viewer with their beauty and complexity. Escher's exploration of spirals in his art not only showcases his artistic talent but also his fascination with the mathematical properties of these forms. In addition to his renowned spiral designs, Escher also delved into the world of wallpaper group designs. These patterns are created by repeating geometric shapes in a specific arrangement, resulting in visually striking and intricate designs. Escher's exploration of wallpaper group designs demonstrates his ability to combine art and mathematics, creating visually appealing works that also possess deep mathematical significance [2-3].

Escher's work has had a profound impact on both the art and mathematics communities, inspiring generations of artists and mathematicians to explore the intersections between these fields. His unique approach to art, which combines mathematical principles with visually stunning and thought-provoking designs, has made his work a lasting influence on the creative landscape. In contemporary culture, Escher's influence extends far beyond the art world, with his designs and concepts being applied in various fields such as architecture, design, and even technology. For example, the intricate patterns and structures found in Escher's work have inspired the design of modern buildings and public spaces, while his exploration of self-referential artwork has contributed to the development of artificial intelligence and virtual reality technologies.

M.C. Escher's life and work have left a legacy on both the art and mathematics communities. His innovative approach to combining mathematical principles with art has resulted in iconic works like *Gallery*, and his exploration of spiral designs and wallpaper group designs has captivated and inspired generations. Escher's impact extends beyond the art world, with his designs and concepts influencing various fields and continuing to be a source of fascination and inspiration for many [4].

Inspired by *Gallery*, in this paper we first analyze its structure. Then, we establish an automatic method to create Escher-like spiral patterns like *Gallery*, which can be used in decorative fields.

2 MATHEMATICAL STRUCTURE OF GALLERY

M.C. Escher's *Gallery* [4] is an iconic lithograph that exemplifies the artist's mastery of visual illusion and mathematical concepts. Created in 1956, this artwork features a complex, self-referential scene that invites viewers to ponder the nature of perception and human experience. In *Gallery* Escher depicts a gallery of paintings that appears to be tilting forward, with the viewer standing at the bottom looking up. The ceiling of the gallery is a series of angled arches, and the floor appears to be a grid pattern that extends into the distance. The most striking feature of the artwork is the presence of a small figure in the lower left corner, who is looking through a spyglass at a painting that is hanging on the wall in the upper right corner. This painting, in turn, depicts the gallery from an elevated perspective, with the same figure standing in the lower left corner, holding the spyglass.

The self-referential nature of *Gallery* creates a sense of infinite recursion, as the viewer is constantly drawn back and forth between the two-dimensional surface of the lithograph and the three-dimensional space depicted within it. This visual exploration of space, symmetry, and infinity is a testament to Escher's deep understanding of mathematical concepts and his ability to incorporate them into his art.

One of the key mathematical principles at play in *Gallery* is the concept of projective geometry, which deals with the relationship between points, lines, and planes in space. Escher's use of angled arches and a grid-patterned floor creates a sense of depth and perspective, while the self-referential nature of the artwork challenges our understanding of reality and illusion. Another mathematical concept that Escher explores in *Gallery* is the idea of tessellation, or the repetition of geometric shapes to create a pattern that covers a surface without gaps or overlaps. The grid-patterned floor and the angled arches of the ceiling can be seen as tessellations, while the overall composition of the artwork can be viewed as a complex tessellation of shapes and spaces [5-6].

Overall, M.C. Escher's *Gallery* is a masterpiece of visual art that combines mathematical principles with artistic innovation to create a work that is both aesthetically pleasing and intellectually stimulating. Its impact on both the art and mathematics communities has been profound, inspiring generations of artists and mathematicians to explore the intersections between these fields and to push the boundaries of what is possible in both domains.

3 GRIDS AND COMPLEX PERIODICITY IN GALLERY

At the heart of M.C. Escher's *Gallery* lies the ingenious Escher grid, a sophisticated method that enables the creation of an illusion of an infinite loop, where the boundaries between reality and illusion blur seamlessly. This grid serves as the foundational structure upon which Escher builds his intricate visual masterpiece, demonstrating his profound understanding of both mathematics and art.

The Escher grid is a remarkable hybrid of linear and curvilinear worlds, where complex transformations occur in a manner that preserves the integrity and coherence of the image. In the linear world, Escher employs a meticulous approach to each rotation and scaling operation, ensuring that the transitions between different elements of the artwork are continuous and harmonious. This precision allows the viewer to perceive the artwork as a cohesive whole, despite the intricate and seemingly paradoxical nature of its construction. To achieve the desired effect of an infinite loop, Escher employed a technique involving arc-shaped grids, which distinguishes his work from more conventional grid systems. Unlike straight grids, which can become rigid and unnatural during transformation, arc-shaped images allow for smooth bending and expansion. This flexibility ensures that the image remains fluid and organic, even as it undergoes complex transformations that would typically result in distortion or loss of coherence.

The concept of complex periodicity is central to understanding *Gallery*. A complex period γ is defined as a value that, when applied repeatedly, results in the original image. In Escher's work, this periodicity is achieved through a combination of rotations and scaling. Let g be a function defined on the complex plane C, taking values in {black, white}. For every point $z \in C$, the color assigned to z is g(z). Since *Gallery* is a periodic image with period γ, we have

$$g(\omega)=g(\gamma\omega).$$

To find the relationship between the rotation factor of 256 and the rotation period γ, Escher introduced a scalar α on the complex plane C*. By mapping the line graph back to a dual-periodic graph using the exponential function, Escher was able to create a new image that remains invariant under translation

$$L_{256} = Z2\pi i + Zlog256.$$

This approach shows Escher's deep understanding of complex analysis and his ability to apply mathematical concepts to create visually stunning artwork. The use of complex periodicity and dual-periodic graphs not only adds to the aesthetic appeal of *Gallery* but also highlights the intricate connections between mathematics and art. Moreover, the Escher grid serves as a powerful tool for exploring the relationship between mathematics and art. Through his use of this grid, Escher demonstrates how mathematical concepts can be employed to create visually appealing and conceptually rich artworks. This has had a lasting impact on both the art and mathematics communities, inspiring generations of artists and mathematicians to explore the intersections between these fields and to push the boundaries of what is possible in both domains. In Fig 1, we show the process of grid construction used in creating *Gallery* [1].

Escher grid and the concept of complex periodicity are critical components of M.C. Escher's *Gallery* enabling the creation of an illusion of an infinite loop that captivates and intrigues viewers. His innovative use of mathematical principles and artistic techniques showcases his mastery of both disciplines, making *Gallery* a timeless masterpiece that continues to inspire and challenge us today.

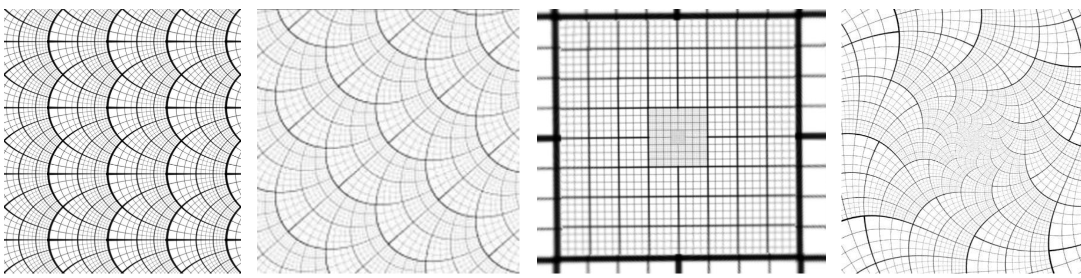

Figure 1 Left to Right: The Four Construction Steps of the Conformal Grids Used in *Gallery*

4 CONFORMAL MAPPING AND WALLPAPER GROUP IN GALLERY

Conformal mapping[7-8], a mathematical technique that preserves angles locally, plays a pivotal role in the creation of M.C. Escher's masterpieces, including *Gallery*. By mapping the plane onto the complex plane and applying conformal transformations, Escher was able to fashion images that are not only mathematically rigorous but also possess a remarkable aesthetic appeal. In *Gallery* Escher's use of conformal mapping is particularly noteworthy, as it ensures that the angles between curves remain unchanged, even as the shapes themselves undergo intricate transformations. This property is crucial for maintaining the illusion of continuity and coherence throughout the artwork, allowing viewers to perceive complex patterns and structures as if they were part of a single, cohesive whole.

Figure 2 Left to Right: Basic Wallpaper Pattern, Spiral Pattern Similar to *Gallery*, and Spiral Pattern with more Intense Spiral Scale

Wallpaper groups [9-11], also known as plane crystallographic groups or plane symmetry groups, are mathematical structures that describe the symmetries of patterns that repeat infinitely in two dimensions. These groups consist of combinations of translations, rotations, reflections, and glide reflections, and they play a significant role in Escher's work. In *Gallery* Escher employs several wallpaper group symmetries to create a sense of depth and complexity that is both visually striking and mathematically precise. The primary symmetries used are translations and rotations, which work in harmony to produce a composition that is both stunning and meticulously crafted. Fig 2 shows two spiral patterns like *Gallery* and the wallpaper motif used.

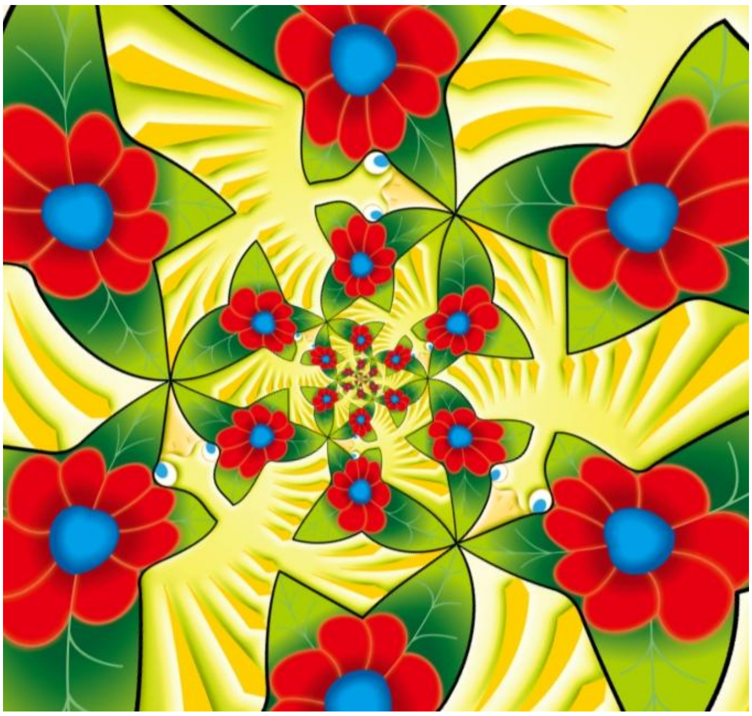

Figure 3 Flower-Bird Spiral Pattern Similar to *Gallery*

Translations involve shifting the entire pattern by a fixed vector, and in *Gallery*, they are employed to create the illusion of an endless corridor. Repeated motifs of doors and windows appear at regular intervals, creating a sense of movement and continuity that draws the viewer into the artwork. This use of translation symmetry not only enhances the visual appeal of the piece but also reinforces the illusion of an infinite loop that is characteristic of Escher's style. Rotations

involve rotating the pattern around a central point, and in *Gallery*, Escher uses rotations to introduce a sense of dynamism and symmetry into the composition. Certain elements within the image rotate around specific points, creating a harmonious and balanced arrangement that is both pleasing to the eye and mathematically sound. In Figs 3-4, based on the method introduced, we display show two Escher-like patterns similar to *Gallery*.
The combination of conformal mapping and wallpaper group symmetries in *Gallery* showcases Escher's deep understanding of mathematical principles and his ability to apply them to create visually stunning artworks [12]. This fusion of art and mathematics is a hallmark of Escher's style and has had a lasting impact on both fields, inspiring generations of artists and mathematicians to explore the intersections between these disciplines. Moreover, the use of these mathematical techniques in *Gallery* serves as a powerful testament to the potential for art to convey complex ideas and concepts in a way that is accessible and engaging to a wide audience. Escher's ability to blend mathematical rigor with artistic innovation has made his work a timeless classic that continues to resonate with viewers today.

Figure 4 Flower Spiral Pattern Similar to *Gallery*

Conformal mapping and wallpaper group symmetries are essential components of M.C. Escher's *Gallery* enabling the creation of an artwork that is both mathematically sophisticated and aesthetically captivating. The innovative use of these mathematical techniques showcases Escher's mastery of both disciplines and serves as an enduring example of the potential for art to inspire and challenge our understanding of the world around us.

5 CONCLUSION

M.C. Escher's *Gallery* is a masterpiece that beautifully illustrates the intersection of art and mathematics. Using complex transformations and wallpaper group symmetries, Escher created a work of art that challenges our perception of reality and invites us to explore the hidden beauty of the mathematical world. M.C. Escher's work transcends mere displays of mathematical prowess; it represents a profound creative exploration of the intricate relationship between art and mathematics. By skillfully incorporating complex transformations and wallpaper group symmetries, Escher was able to craft images that are not only visually stunning but also intellectually stimulating, inviting viewers to delve into the layers of meaning and mathematical elegance embedded within his creations.
One of the most captivating and striking features of *the Gallery* is the so-called *Bird Effect*, where birds appear to transform seamlessly into flowers as they move through the image. This remarkable effect is achieved through a sophisticated combination of rotations, scalings, and translations, which collectively create a sense of metamorphosis and continuity that is both enchanting and thought-provoking. The bird-to-flower transformation is not merely a visual trick but a testament to Escher's ability to harness mathematical principles to serve his artistic vision, resulting in an image that challenges our perceptions and invites us to explore the boundaries of what is possible in the realm of visual representation.
In addition to the *Bird Effect*, *Gallery* is adorned with intricate patterns of flowers and leaves that further exemplify Escher's mastery of mathematical principles in art. These patterns are meticulously created using similar mathematical concepts, with each element strategically positioned to maintain the overall symmetry and coherence of the image. The careful arrangement of these elements not only enhances the aesthetic appeal of the artwork but also reinforces the underlying mathematical structure, creating a harmonious balance between form and content. Escher's use of complex transformations and wallpaper group symmetries in *Gallery* goes beyond mere technical proficiency; it reflects a deep

philosophical inquiry into the nature of reality and the ways in which we perceive and interpret the world around us. By pushing the boundaries of traditional artistic techniques and mathematical applications, Escher invites viewers to reconsider their understanding of space, form, and the interplay between the two.

Escher's work serves as a bridge between the worlds of art and mathematics, demonstrating that these two seemingly distinct fields can intersect in profound and meaningful ways. His creations inspire artists to explore mathematical concepts as a source of inspiration and creativity, while also encouraging mathematicians to seek out the aesthetic beauty inherent in their work. In essence, M.C. Escher's *Gallery* stands as a testament to the power of creative exploration and the unifying force of mathematical principles in art. It is a work that not only dazzles the eye but also stimulates the mind, inviting viewers to embark on a journey of discovery and appreciation for the intricate dance between art and mathematics. Moreover, the enduring legacy of Escher's work lies in its ability to transcend time and cultural boundaries, speaking to a universal human desire to understand and appreciate the beauty and complexity of the world around us. *Gallery*, with its rich tapestry of mathematical concepts and artistic innovation, remains a timeless masterpiece that continues to captivate and inspire generations of viewers, offering endless possibilities for exploration, interpretation, and admiration. Escher's legacy is not just in his art; it is in his ability to inspire us to see the world through a different lens. By blending art and mathematics, Escher showed us that these two fields are not only related but also complementary. His work continues to inspire artists, mathematicians, and scientists, encouraging us to explore the endless possibilities that lie at the intersection of these disciplines.

COMPETING INTERESTS

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