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CONVERTIBLE BOND PRICING BASED ON MONTE CARLO SIMULATION

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Abstract: Convertible bonds, as a novel financing instrument, possess dual characteristics of both conventional bonds and options due to the inclusion of clauses with American option-like features such as conversion, redemption, putback, and conversion price adjustment. Therefore, the pricing of convertible bonds is of considerable importance. This study has gathered a substantial amount of relevant industry data. For accuracy considerations, this paper employs the Random Forest algorithm and LightGBM algorithm to predict the probability and price of triggering downward adjustment clauses, achieving a classification accuracy of 60.162% and a regression goodness of fit of 0.757. Subsequently, Monte Carlo simulation was utilized for convertible bond pricing prediction, resulting in the calculation of MAPE values for two convertible bonds of 5.15% and 9.6%, respectively, indicating the model's high accuracy. Finally, a financial analysis of the results was conducted to provide investment recommendations. Leveraging this research outcome, convertible bonds can be better applied and promoted, and the development of the convertible bond market can increase the proportion of debt financing in the capital market, enabling enterprises to flexibly adjust their capital structure. Its development also provides investors with more investment options.
Keywords: Convertible bonds; Random Forest; LightGBM; Monte Carlo simulation

1 INTRODUCTION

Convertible bonds are bonds that allow the holder to convert the bond into the company's common stock at a price agreed upon at the time of issuance. The interest rate on these bonds is generally lower than that of ordinary corporate bonds, and issuing convertible bonds can reduce the financing costs for companies. Holders of convertible bonds also have the right to sell the bonds back to the issuer under certain conditions, and the issuer has the right to compulsorily redeem the bonds under certain conditions. The Monte Carlo method is a numerical calculation method based on probability and statistical theory. It solves deterministic problems by performing a large number of random simulations to obtain numerical solutions. The Monte Carlo pricing method estimates the price of convertible bonds by simulating random variables. It generates a series of random paths to simulate the changes in convertible bond prices and calculates the bond price on each path. Today, the Monte Carlo method has become an effective method for bond valuation.

Xiao Jie[1] used the pricing method of the binary tree model to conduct empirical analysis on the theoretical value of the bond under various terms. Wang Pu[2] established a convertible bond pricing model with Hull-White stochastic volatility under the sub-fractional jump-diffusion process, and obtained the model parameters using the maximum likelihood function method. Jiao Dian[3] first used the present value of future cash flows method to price the bond value of convertible bonds, then used the traditional B-S model to price the option value of convertible bonds issued from 2015 to 2020 that meet the assumptions of the B-S pricing model, and finally used the extended B-S model with additional terms to conduct a secondary pricing of the convertible bond sample data. Hu Miao[4] used the sliding window method to estimate the upper and lower variances, simulated the underlying asset price using the relevant theories and definitions of G-Brownian motion, combined with the Monte Carlo method to simulate the price of the embedded call option of the convertible bond, and further priced the convertible bond. Du Dongxu[5] derived and constructed a convertible bond valuation model based on multiple linear regression and polynomial characteristics for convertible bond pricing. Li Zaiqiao[6] studied the pricing of convertible bonds based on the Black-Sholes model. Liu Sicheng[7] used the Ada Boost algorithm to price convertible bonds. Wang Yintian[8] adopted the convertible bond pricing model proposed by Tsiveriotis and Fernandes, rigorously incorporating the downward modification clause, as well as the redemption, put, and conversion clauses. She analyzed the impact of the downward modification clause on the pricing of convertible bonds.

Currently, scholars seldom employ the Monte Carlo method for the valuation of domestic convertible bonds, and there are scant studies that provide corresponding recommendations to investors based on the results.

Therefore, This study innovatively introduces downside clauses through the application of machine learning, compares various machine learning models, employs Monte Carlo simulation to price convertible bonds, and offers investment recommendations tailored to specific companies. Consequently, this study possesses certain practical value in the field of convertible bond pricing and investment in China.

2 PREDICTION OF DOWNWARD-ADJUSTMENT CLAUSES

To enhance the innovativeness of the model, data on downward revisions was collected. This study queried the Shanghai Stock Exchange and Shenzhen Stock Exchange for downward revision information of listed companies over the past decade, encompassing issuance size, GICS primary industry codes, and convertible bond rating codes. The specific datasets for this article is sourced from www.sse.com.cn and www.szce.cn. Since the objective is to predict the conversion price following the triggering of downward revision clauses, cases that underwent downward revisions were selected from the datasets. Furthermore, the features "GICS primary industry" and "convertible bond rating" were encoded, with the coding scheme being "1, 2, 3, 4, 5, 6, 7, 8, 9" for "Industrial, Raw Materials, Consumer Goods, Information Technology, Healthcare, Telecommunications, Non-Consumer Goods, Utilities, Finance," respectively. The codes for "CC, B-, A, A+, AA-, AA, AA+, AAA" were sequentially assigned as "1, 2, 3, 4, 5, 6, 7, 8," and the downward revision passing flag was set to 1, while the failing flag was set to 0.

The prediction of conversion prices essentially involves a regression task, where the features include the pre-revision price, issuance size, GICS, and convertible bond rating. Considering accuracy, multiple machine learning models were established: linear regression, support vector machine regression, random forest regression, XGBoost regression, and LightGBM regression. The optimal model was selected based on the evaluation metrics mean squared error (MSE) and R-squared (R2) goodness of fit. After experimentation, the LightGBM model was ultimately chosen for the regression task, with an MSE of 44.616 and an R-squared of 0.757. The model demonstrated strong applicability with only four features. The regression curve for the test set is plotted as shown in Figure 1.

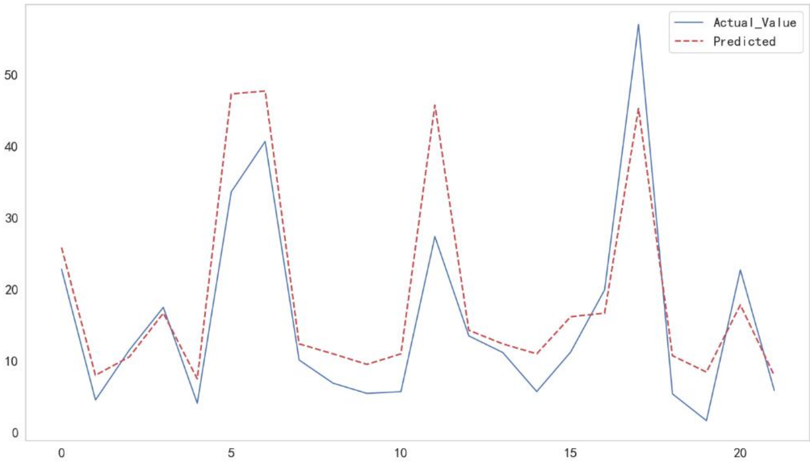

Figure 1 Results of the Regression Curve

Subsequently, a feature importance analysis was conducted. The result graph from the interpretable machine learning SHAP module reveals that the importance ranking is as follows: pre-revision price, issuance size, GICS, and convertible bond rating [9]. Furthermore, the mechanism of specific effects can be discerned. Taking the issuance size as an example, when its value is smaller, the corresponding model output is also smaller. Conversely, as its value increases, the model output also increases. The SHAP results are illustrated in Figure 2.

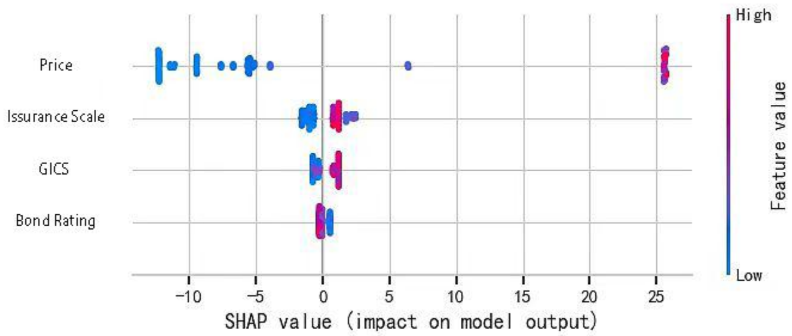

Figure 2 Result of the SHAP

The prediction of whether a downward revision will be approved is essentially a binary classification task, where category 1 represents approval of the downward revision and category 0 represents disapproval. The features remain the pre-revision price, issuance size, GICS, and convertible bond rating. A random forest model is established for classification, which is an ensemble model integrating multiple decision trees [10]. By solving and outputting the number of instances classified into two categories each time, the probability of whether the lower bound adjustment is successful can be calculated. The calculation method is as follows:

$$P=\frac{\sum_{i=1}^{n} kind=1}{\sum_{i=1}^{n} kind=0+\sum_{i=1}^{n} kind=1} \tag{1}$$

This study employed a 10-fold cross-validation method, deriving the final probability of downward revision equal to 0.27 by calculating the average. The model accuracy stood at 60.162%, which is visualized through the confusion matrix depicted in Figure 3.

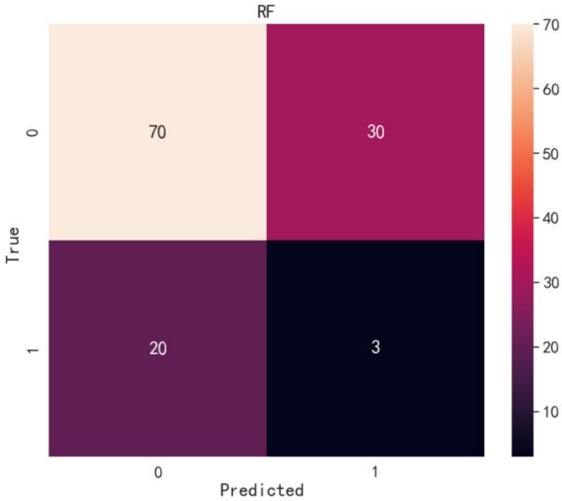

Figure 3 Confusion Matrix

3 MONTE CARLO-BASED PRICING OF BONDS

This article conducts valuation for each bond in China on every trading day. It simultaneously considers the conversion clauses, redemption clauses, putback clauses, and downward revision clauses of each convertible bond, performs 5,000 Monte Carlo simulations, calculates the expected returns under various simulation paths, and ultimately derives the model pricing of the convertible bond for the current day through discounted averaging. Through a large number of simulation paths, the model is able to capture multiple possible trends in stock prices, enhancing the accuracy of pricing. The basic information of the convertible bond is inputted, including the maturity date, conversion start date, coupon rate, redemption price, redemption trigger price, putback trigger price, conversion price, etc. The geometric Brownian motion model is used to simulate the stock price path. Where Z is a standard normal distribution random variable, St represents the asset price at time t, S0 represents the asset price at the valuation time, and σ represents the volatility of the asset price at time t. The stock price itself follows a lognormal distribution.

$$S_t=S_0*\exp\left((r-\frac{1}{2}\sigma^2)t+\sigma\sqrt{t}\cdot Z\right) \tag{2}$$

Utilizing Python to develop a program, a simulated path is created using a random number generator. The mathematical formula for the pricing of convertible bonds is as follows, from which the predictive price of convertible bonds can be derived. Some of the results are presented in Table 1.

$$PredictPrice=\frac{\sum_{i=0}^{5000} price_i}{5000} \tag{3}$$

Table 1 Monte Carlo Pricing Results for Convertible Bonds

Date	Bond_Name	Prediction	Actual
2024-01-26	123087	112.93	115.22
2024-01-25	123087	114.02	115.04
2024-01-24	123087	123.85	112.42
2024-01-23	123087	112.41	112.81
2024-01-22	123087	111.93	114.10
2024-01-19	123087	113.07	115.78
2024-01-18	123087	109.14	115.60
2024-01-17	123087	111.06	116.05

The pricing results of the two convertible bonds are visualized in Figures 4 and 5. To verify the accuracy of the model, this paper adopts the Mean Absolute Percentage Error (MAPE) as an evaluation metric. The calculation formula is as follows: where N represents the data volume, yi denotes the actual value at the i-th observation point, and represents the model's predicted value at the i-th point. A MAPE value of 0% indicates a perfect model, while a MAPE value greater than 100% indicates a poor model. Upon calculation, the MAPE values for the two convertible bonds, "Mingdian Convertible Bond" and "Shengtang Convertible Bond", are 5.15% and 9.6%, respectively, demonstrating the high accuracy of the model.

$$MAPE = \frac{100\%}{N} \sum_{i=1}^{n} \left| \frac{\hat{y}_i - y_i}{y_i} \right| \tag{4}$$

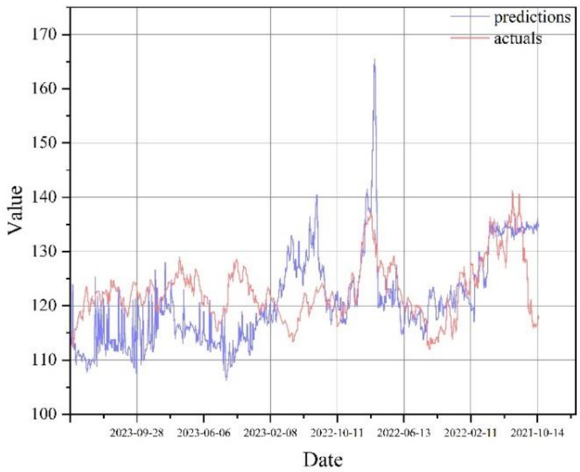

Figure 4 Comparison of Predicted and Actual Values for Mingdian Convertible Bonds

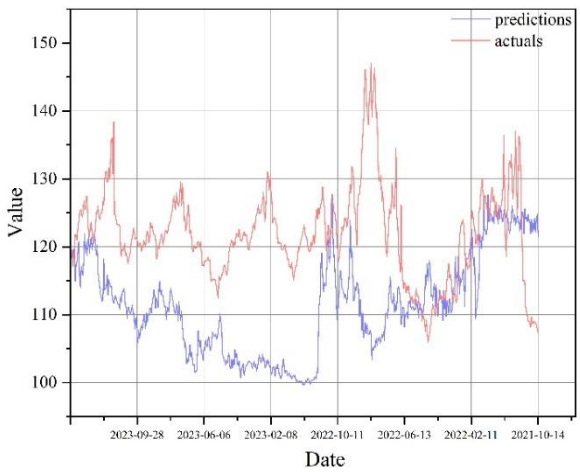

Figure 5 Comparison of Predicted and Actual Values for Shengtang Convertible Bonds

4 FINANCIAL ANALYSIS AND INVESTMENT ADVICE

After utilizing Monte Carlo simulation for the pricing of convertible bonds, this study selected a specific company for further financial analysis. Recent trading data reveals a certain degree of volatility in the price of Mingdian Convertible Bonds. From August 2023 to late January 2024, its price fluctuated between 119 yuan and 125 yuan, followed by a significant decline in February, with the lowest price falling below 113 yuan. Under the influence of fluctuations in the overall economic environment and policy changes, the stock prices of many technology companies have experienced ups and downs, and Mingdian is no exception. Although it maintained stability in the early stages, it has recently shown a clear downward trend, with indicators such as trading volume and turnover rate indicating market unease. The stock market itself is highly volatile, especially during periods when large institutional investors adjust their positions and liquidity is tight, which can lead to rapid price fluctuations. Specifically regarding information development, its stock price is influenced by various factors. For example, recent industry news, the performance of the company's financial reports, and external economic data may all impact investors' psychology. The stock market itself is highly volatile, especially during periods when large institutional investors adjust their positions and liquidity is tight, which can lead to rapid price fluctuations. Specifically regarding information development, its stock price is influenced by various factors. For example, recent industry news, the performance of the company's financial reports, and external economic data may all impact investors' psychology.

Investors can utilize this model to predict the trend of company stock prices, select appropriate arbitrage methods, and subsequently profit. Investors may prioritize the GICS primary sectors, as companies in these sectors often possess a relatively stable business model and strong profitability, which reduces the risk of corporate default, thereby rendering the convertible bonds issued by them safer. Furthermore, certain categories within the primary sectors, such as consumer staples and healthcare, exhibit a certain degree of resilience against economic cyclicality. Even during economic downturns, the performance of these sectors remains relatively stable, thus bolstering the value of convertible bonds. Additionally, investors may endeavor to select companies with high ratings, as a high rating signifies a lower likelihood of corporate default. Consequently, investors can hold these convertible bonds with greater peace of mind, without fearing that the company may be unable to repay the debt or interest, leading to a more stable stock price and, consequently, more accurate model predictions.

5 CONCLUSIONS

This article addresses the pricing issue of convertible bonds, employing Monte Carlo simulation based on the least squares method for pricing prediction. It simultaneously considers the downward adjustment clause, as well as the redemption, putback, and conversion clauses. Furthermore, it utilizes random forest and LightGBM algorithms for predicting the downward adjustment price and probability. The accuracy of regression and classification tests has been found to be high. Monte Carlo simulations were conducted for related industries, and the MAPE values for the two tested convertible bonds were 5.15% and 9.6%, respectively. This demonstrates the strong practicality of the Monte Carlo method adopted in this article for pricing convertible bonds. For investors, they can choose appropriate arbitrage methods based on the predicted pricing of convertible bonds. In future research, further consideration could be given to incorporating credit risk, adjusting the model structure, and enhancing model accuracy, thereby enabling better decision-making for both companies and investors.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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