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DESIGN OF TRAM ALGORITHM BASED ON GRAPH TRANSFORMATION

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Abstract: In this study, the TRAM (Trigonometric Reduction and Acceleration for Matrices) algorithm, based on small-angle approximation, table lookup, and matrix compression, is proposed to address the arithmetic bottleneck and real-time constraints of the ARM Cortex-M4 processor in laser graphics processing. The small-angle approximation replaces the generalized trigonometric function on floating point; the double-indexed table is designed to reduce the trigonometric function storage; and the matrix compression strategy is proposed to reduce the computational complexity. The experimental results show that the computation time of the algorithm for a single graphical transformation is significantly shorter than that of the traditional table lookup method and a single hardware acceleration strategy under a 240MHz main frequency. At the same time, the computation error is always kept lower than 0.035% so that the scanning system can realize a higher laser scanning frame rate to meet the real-time requirement of industrial laser dynamic projection and provide a reusable methodological framework for the real-time graphical processing of the resource-constrained embedded system. It provides a reusable methodological framework for real-time graphics processing in resource-constrained embedded systems, which is of great applied value.
Keywords: Graphics transform; ARM; Algorithm design; DMX512 protocol

1 INTRODUCTION

Since the 1990s, laser scanning technology has experienced rapid development in the field of stage performance applications, from simple static beam display to complex dynamic visual effects, and has become an important part of modern visual effects [1]. Laser scanning technology, through the integration of digital signal processing, embedded systems, and computer graphics processing technologies, projects complex patterns, texts, and dynamic animations at a response speed of milliseconds and develops in a more intelligent and universal direction.
Despite the rapid progress, ARM-based embedded platforms are widely adopted due to their low power consumption (usually 5-10W) and compact appearance, but embedded platforms such as ARM show insufficient computing power for laser graphics processing calculations and cannot meet the real-time requirements of laser graphics transformation rendering [2]. This study innovatively proposed a TRAM algorithm based on graphics transformation, which reduces the calculation time of laser graphics transformation in the ARM Cortex-M4 series chips to less than a few microseconds by introducing small angle approximation, table lookup, and matrix compression through hardware acceleration and algorithm reconstruction. This further improves the real-time performance of the laser scanning system and realizes high frame rate laser image transformation.

2 THE UTILIZATION OF COMPUTER GRAPHICS IN LASER SCANNING SYSTEMS

Laser scanning system through the multi-color laser beam in the field of view of the dynamic transformation of the combination of the plane to achieve smooth animation effects and complex art forms [3], laser image transformation of the mathematical basis of computer graphics from the geometric transformation of the theoretical system. This section takes laser vector graphics in three-dimensional chi-square coordinates as the research object [4] and systematically analyzes the mathematical model and realization path of geometric transformation in laser scanning technology.
The expression of computer graphics in graphical transformations, such as rotation, movement, and scaling, typically utilizes a matrix. The rotation matrix is denoted by $R(\theta)$ the translation matrix by T, and the scaling matrix by S. The original coordinate points on the three-dimensional coordinate system are represented by $P(X,Y,Z)$, while the transformed coordinates of the operation are designated by $P^{'}(X^{'},Y^{'},Z^{'})$. The following formulae (1) -(3) represent the laser point after the translation transformation, rotation, and scaling, respectively, after the transformation of the matrix representation.

$$P' = T * P = \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \tag{1}$$

In Equation(1), $t_x/t_y/t_z$ represent the translations of the laser point in the X/Y/Z directions, respectively.

$$P' = R_y(\theta) * P = \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \tag{2}$$

In Equation(2), the rotation of the graph around the Y-axis is taken as an example.

$$P' = S * P = \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \tag{3}$$

In Equation(3), $S_x/S_y/S_z$ represents the scaling factor of the laser point in X/Y/Z direction, respectively.

In the process of actual laser image transformation, it is often the case that a single transformation matrix is inadequate in achieving the desired transformation effect. To illustrate, when an arbitrary point on the axis is to be rotated, the center of rotation must first be determined, and the image must be translated to the origin [5]. Subsequently, the image must be moved back to the center after completing the rotation. Therefore, it is necessary to merge multiple transformation matrices into a single composite matrix to reduce the computational complexity.

3 THE TRAM ALGORITHM IS PREDICATED ON THE TRANSFORMATION OF GRAPHS

The transformation of laser graphics frequently involves multiple transformations concurrently; however, the method of reducing the computational complexity by combining multiple transformation matrices into a single composite matrix does not result in a high response speed for the laser scanning system. Furthermore, the frequent trigonometric operations pose a challenge to the ARM processor.

This study proposes a graphical transformation-based TRAM (Trigonometric Reduction and Acceleration for Matrices) algorithm design that combines small-angle approximation, look-up table method (LUT), and matrix compression to meet the high-speed response of graphical transformation. Algorithm-specific design ideas:

3.1 Small Angle Approximation

Divide the circle into *n* equal parts

$$n = 360^0 / \theta \tag{4}$$

The region of the triangle subsequent to n equal divisions, as illustrated in Figure 1, is denoted by S_1.

$$S_1 = (r^2 \sin\theta) / 2 \tag{5}$$

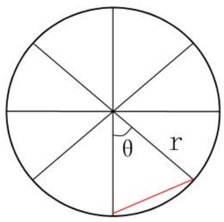


Figure 1 Schematic Diagram of Circular Calculated Area

It is evident that, as the value of n, referred to as the number of circular equivalents, increases, there is a corresponding increase in the degree to which the aggregate area of the *n* isosceles polygons approaches the area of the circle.

$$S_1 * n = (r^2 \sin\theta) * n / 2 = (r^2 \sin\theta) * (360^0 / \theta) / 2 = \pi r^2 \tag{6}$$

The limit of Equation(6) is given by

$$\lim_{\theta \to 0} (\sin\theta / \theta) = \pi / 180^\circ \tag{7}$$

Therefore, as the value of θ approaches zero,

$$\sin\theta \approx \theta \tag{8}$$

3.2 Look-up Table Method (LUT)

Firstly, all the index values in the sin function are mapped to the range of 0 to 90 degrees through the trigonometric algorithm, and the cos function is equivalently converted to the sin function; secondly, a table of values of 0 to 90 degrees is set up for every 10 indexes of the sin function and every 1 index of the cos function, and the corresponding numerical values are stored in the table as shown in Table 1 respectively.

Table 1 Trigonometric Functions Table of Values

Degrees	Sin Function	Degrees	Cos function
0	0.0	0	1.0
10	0.17364817766693034885171662676931	1	0.99984769515639123915701155881391
20	0.34202014332566873304409961468226	2	0.99939082701909573000624344004393
30	0.5	3	0.99862953475457387378449205843944
40	0.64278760968653932632264340990726	4	0.99756405025982424761316268064426
50	0.76604444311897803520239265055542	5	0.99619469809174553229501040247389
60	0.86602540378443864676372317075294	6	0.99452189536827333692269194498057
70	0.93969262078590838405410927732473	7	0.99254615164132203498006158933058
80	0.98480775301220805936674302458952	8	0.99026806874157031508377486734485
90	1.0	9	0.98768834059513772619000247693 44

When Equations(8) are combined with Table 1, the result is

$$\begin{aligned} \sin 72.5^\circ &= \sin 70^\circ \cos 2.5^\circ + \cos 70^\circ \sin 2.5^\circ \\ &\sin 70^\circ \cos 2^\circ + \sin 20^\circ * 2.5^\circ * \pi / 180^\circ \\ &= 0.95404362945391005654471613227652 \end{aligned} \tag{9}$$

The following are theoretical values for trigonometric calculations.

$$\sin 72.5^\circ = 0.95371695074822692114384706460026 \tag{10}$$

According to the Equations (9) and Equations (10), when compared to the calculation error of 0.034245%, it can be concluded that the small angle approximation to the table method, when combined with the calculation results of the error, is sufficiently small. The algorithm in this part of the content has effectively reduced the amount of data operations and has ensured the validity of the data.

3.3 Matrix Compression

In accordance with the rotational symmetry of the matrix, the following equation is derived:

$$R_y(-\theta) = R_y^T(\theta) \tag{11}$$

According to Equations (11), a matrix with a rotation angle of is obtained by taking the transpose of without storing a new matrix additionally.

As illustrated in Table 2, a comparative analysis was conducted to assess the computational efficiency of various schemes on the ARM processor (GD32F470VIT6) with a 240 MHz main frequency. The analysis involved the transformation of laser patterns using the multinomial matrix, a process that was executed under different computational schemes. The experimental results demonstrated that the TRAM algorithm, which was proposed in this study, exhibited a significant enhancement in computation speed, achieving an improvement of 99.87% compared to other tested schemes.

Table 2 Calculation Time for Different Calculation Schemes

Calculation Scenarios	Calculation Time(us)
Table lookup method	59998
DSP core enabled	6003
TRAM algorithm	8

In summary, the TRAM algorithm synthesizes the merits of small-angle approximation, Table lookup, and matrix compression, thereby enhancing the efficiency of data processing in graphical transformations. Concurrently, it ensures that data error is meticulously regulated within the range of 0.035%. Refer to Table 3, which demonstrates that the algorithm exhibits neglect computational error.

Table 3 TRAM Calculation Comparison Table

Sin Function(°)	Theoretical Value	TRAM Algorithm Calculates the Value	Error(%)
12.5	0.216440	0.216513	0.033727
22.5	0.382683	0.382814	0.034231
32.5	0.537300	0.537483	0.034059
42.5	0.675590	0.675821	0.034192
52.5	0.793353	0.793625	0.034284

62.5	0.887010	0.887314	0.034272
72.5	0.953717	0.954043	0.034245

4 THE TRAM ALGORITHM IS VALIDATED THROUGH THE IMPLEMENTATION OF GRAPH TRANSFORMATION

As illustrated in Figure 2, the laser control system architecture of the ARM platform is constructed based on the TRAM algorithm and its multi-terminal cooperative work mode. In this physical test platform, the laser scanning system adopts a modularized design architecture, the laser controller interacts with the upper control terminal through a DMX512 cable, and the user flexibly selects the DMX512 professional console, DMX512elf intelligent mobile terminal, and computer software platform according to the needs of the three control schemes.

At the system configuration level, the computer software platform assumes the core parameter configuration function. Through the network line converter and the controller, two-way communication is established to allow for in-depth configuration of laser scanning parameter calibration. At the operational level, a hierarchical control strategy is adopted. The user can either directly call the controller's SD card pre-stored standardized laser graphics library or use the computer software to generate and preview the laser trajectory in real time. Finally, the TRAM real-time control algorithm is transplanted to the ARM Cortex-M4 processor, and the real-time control of laser graphic transformation is realized through the combination of an optimized algorithm kernel and a hardware acceleration unit [6-8].

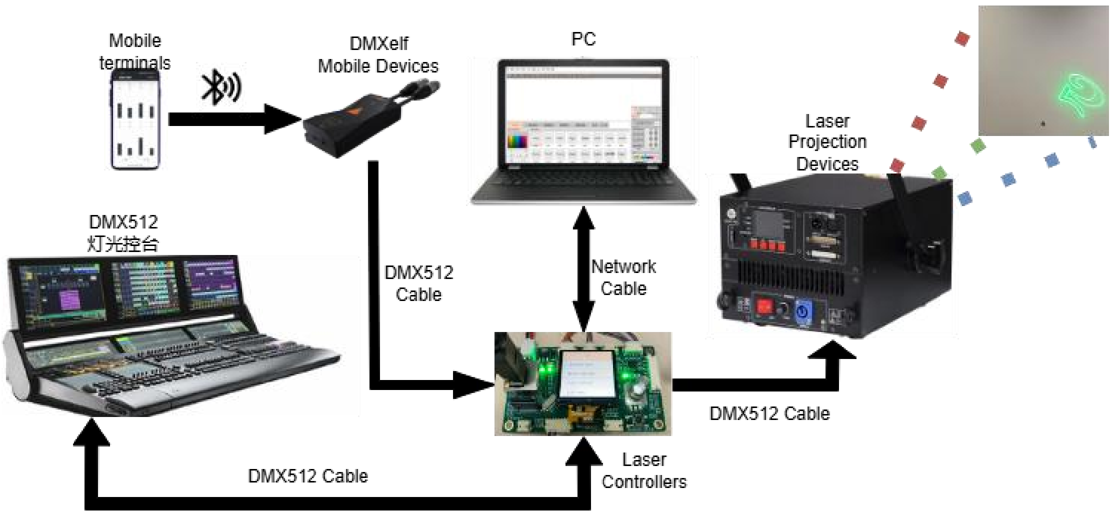
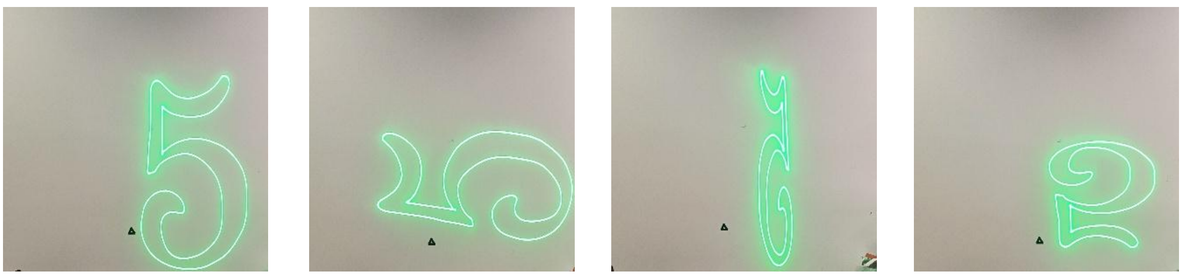


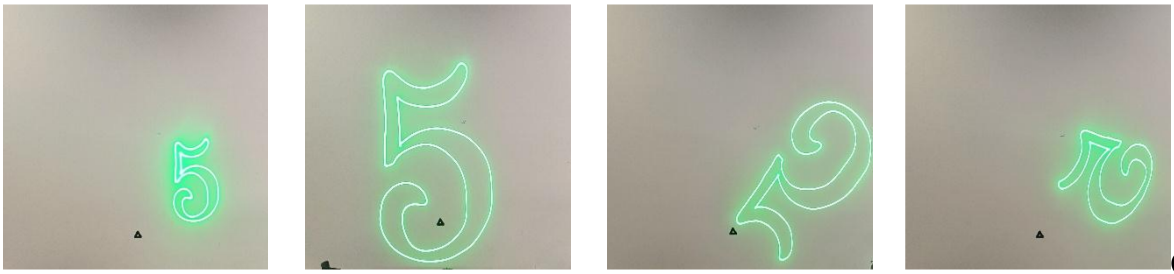
Figure 2 A Platform for Laser Playback that is based on an ARM Controller

Figure 3 demonstrates the effect of laser graphic transformation of the TRAM algorithm ported to an ARM Cortex-M4 processor-based laser controller (240MHz main frequency). The experimental data show that the laser scanning system is able to process laser vector frame map data with more than 800 points. Specifically, the laser scanning system fully meets the real-time requirements of laser graphics transformation with frame rates above 60 FPS, whether it is a single rotational transformation, a translation transformation, a scaling transformation, or a composite transformation.

To further the real-time performance of the algorithm, we invited 20 professional testers to conduct a subjective experience, evaluation criteria: playback smoothness maintenance, color reproduction, image stability and other aspects, the final 20 people's subjective MOS average rating of ⩾ 4.5 (5 is excellent).



(a) Original drawing **(b) Rotation about the Z-axis** **(c) Rotate about the Y-axis** **(d) Rotation about X-axis**



(e) Zoom out **(f) Panning** **(g) Rotate + Zoom + Pan** **(h) Rotate + Zoom + Pan**

Figure 3 Diagram of TRAM Algorithm based on Graph Transformation

5 CONCLUSION

Combining the experimental phenomena and feedback results, this study confirms the significant advantages of the TRAM algorithm based on graphics transformation in laser scanning systems through three levels: theoretical modeling, algorithm optimization, and system validation. The scheme successfully improves the graphics computation efficiency of the ARM processor to 3.5 times that of the traditional method, and the subjective MOS score reaches the professional -level display standard (⩾4.5).

The TRAM algorithm shows excellent real-time performance in the laser scanning system to quickly respond to the user's operation commands, and the graphical transformation process is smooth without any significant delay, providing an excellent overall user experience. The TRAM algorithm based on graphical transformation proposed in this paper effectively reduces the computation time in the ARM processor and significantly improves the real-time performance of the laser system, which provides a solid theoretical support for high-speed laser scanning, and thus makes the laser scanning system show significant advantages in high real-time display and complex graphical processing.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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