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MODELING AND FREQUENCY ESTIMATION OF FLIGHT CYCLE SIGNALS IN COMPLEX ENVIRONMENTS BASED ON THE FAST FOURIER TRANSFORM

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Abstract: Airplanes have now become an important travel tool, and airspeed is the key data to be observed in the process of aircraft flight, therefore, in order to reduce the probability of aircraft stalls and other accidents due to abnormal airspeed data, it is of great significance to accurately measure the airspeed. This study focuses on the frequency estimation of the received signal in flight cycle, combines with the signal processing requirements of laser speed measurement system, constructs a variety of mathematical models and designs the corresponding solution algorithms, and systematically explores the noise characteristics, frequency estimation and intermittent signal processing and other key issues. In this paper, the fast Fourier transform (FFT) algorithm and descriptive statistical analysis of z(t) noise type belongs to Gaussian white noise. The reasonableness and estimation accuracy of the main frequency are verified by comparing the filtered signal with the fitted non-noise signal. This study proposes a frequency estimation method that combines Fast Fourier Transform (FFT) with descriptive statistical analysis. The method effectively suppresses environmental noise interference using an adaptive threshold adjustment based on the Constant False Alarm Rate (CFAR) algorithm. A band-pass filter is employed to eliminate noise, and residual analysis is used to validate the accuracy of frequency estimation. Additionally, an innovative signal reconstruction method under intermittent sampling is proposed, providing theoretical support for frequency estimation and signal processing in complex environments.
Keywords: Fast Fourier transform; Residual analysis; Bandpass filtering; Gaussian white noise; Doppler shifted letter

1 INTRODUCTION

Laser velocimetry is a viable method of measuring air speed. Laser velocimetry systems capture the spectrum of all scattered signals within the sampling bandwidth of the electronic system at a given moment in time through a technique called coherent detection frequency discrimination. These signals include airspeed Doppler-shifted signals, DC noise, relative intensity and phase noise of the laser, and Doppler noise generated by interfering objects [1]. Due to the different frequency components of the interfering signals at different time points, the background noise cannot be simply suppressed. If a fixed detection threshold is used, it is easy to incorrectly detect interfering signals [2]. In particular, the intensity noise and phase noise of lasers, which have a large amount of energy, can easily exceed the detection threshold, thus interfering with the detection results [3]. In order to accurately solve the atmospheric vector signal and accurately detect the airspeed Doppler shifted signal from the signal spectrum, the current particle image velocimetry technique can accurately measure the speed of sound, but it still has defects in filtering the interference signal [4]. In this paper, we design an algorithm to adaptively adjust the detection threshold for laser velocimetry based on the principle of Constant False Alarm (CFAR) detection, which adaptively adjusts the number of reference and protection units and selects the CFAR processing method through the change of the exponential angular coefficient of the detection unit, $S_k = x_D - x_{D-1}$ [5]. In this study, we model and analyze the received signal frequency estimation problem, and the main goal is to design suitable frequency estimation algorithms according to the characteristics of the received data in different flight cycles. The marginal contributions of this study are primarily reflected in four aspects. First, an innovative high-precision frequency estimation method is proposed by combining Fast Fourier Transform (FFT) with descriptive statistical analysis, enabling accurate frequency identification in complex environments. Second, the adaptive Constant False Alarm Rate (CFAR) algorithm is introduced into the laser velocimetry system, effectively addressing the variability of environmental noise and improving the reliability of signal detection. Third, a method combining band-pass filtering and residual analysis is applied to accurately extract valid signals and validate the accuracy of frequency estimation. Finally, a signal reconstruction method under intermittent sampling conditions is proposed, providing a new theoretical basis for signal processing in future complex environments. In conclusion, this study offers innovative theoretical support for laser velocimetry technology and signal processing in dynamic, complex environments.

2 NOISE ANALYSIS BASED ON FOURIER TRANSFORM AND AUTOCORRELATION ANALYSIS

Some basic assumptions are established for the convenience of this study: the noise z(t) is independent in each flight cycle and obeys a specific distribution. The noise characteristics are stable over a flight cycle, i.e., the mean and

variance are constant. The received signal satisfies the expression: $x(t) = Asin(2\pi ft + \varphi) + z(t)$ where A, f, φ denote amplitude, frequency, and phase. The non-noise signal portion (amplitude, frequency, phase) remains stable from flight cycle to flight cycle, but may vary from flight cycle to flight cycle. The sampling interval $T = 2 \times 10^{-9}$ seconds is fixed to satisfy the Nyquist sampling theorem, which states that the signal frequency is less than one-half of the sampling frequency. The data were sampled long enough to support frequency analysis.
In this paper, a fast Fourie transform r (FFT) algorithm is used
Let the sequence of sinusoidal signals y(n) be

$$y(n) = Acos\left(\frac{2\pi f_0 n}{f_s} + \emptyset\right) \tag{1}$$

The rectangular window function w(n) is 1 from n=0 to n=N-1 and 0 at other times. its discrete Fourier transform W(k) is N at k=0 and 0 at other values of k. The discrete Fourier transform W(k) of the rectangular window function is

$$W(k) = \frac{sin\,(\pi k)}{sin\,(\pi k/N)} e^{-j\pi k(N-1)/N} \tag{2}$$

Doing N-point DFT on y(n) yields the discrete spectrum Y(k), due to the symmetry of the real sequence DFT, the effect of negative frequency components can be ignored, and only the first N/2 points of the discrete spectrum are considered, and let Re(x) denote to take the real part of x, then the real part of FFT coefficients Re(Y(k)) is

$$Re\big(Y(k)\big) = \frac{A}{2}\left[\frac{sin\,(\pi(k-k_0))}{sin\,(\pi(k-k_0)} \cdot cos\,(\emptyset - \pi(k-k_0)(N-1)/N)\right] \tag{3}$$

Due to the fence effect of the Discrete Fourier Transform (DFT), if the frequency of the input signal does not correspond exactly to the discrete frequency points of the DFT, the true frequency of the signal may fall between two neighboring DFT frequency lines. In this case, an error occurs when the peak spectral line positions obtained from the Fast Fourier Transform (FFT) are used directly to estimate the signal frequency. The range of this error $[-f_s/(2N), f_s/(2N)]$ is between, where f_s is the sampling frequency and N is the number of samples. Remember that the peak spectral line number in the signal spectrum is m. The peak spectral line Re(Y(k)) and its corresponding phase $\emptyset(m)$ are respectively

$$Re\big(Y(m-1)\big) = \frac{Asin(\pi\delta)}{2\pi(1+\delta)/N} cos\,(\emptyset(m)) \tag{4}$$

$$Re\big(Y(m+1)\big) = -\frac{Asin(\pi\delta)}{2\pi(1-\delta)/N} cos\,(\emptyset(m)) \tag{5}$$

An interpolating polynomial is constructed using the ratio of the real part of the coefficients of the peak spectral line to its neighboring spectral line of the next largest value, and the frequency bias δ estimated by this method is

$$\delta = \begin{cases} \delta_1 = -\frac{Re(Y((m+1))}{\left(Re(Y(m)) - Re(Y(m+1))\right)}, [\delta_1 > 0, \delta_2 > 0] \\ \delta_2 = -\frac{Re(Y((m-1))}{\left(Re(Y(m)) - Re(Y(m-1))\right)}, other \end{cases} \tag{6}$$

The generalized frequency correction formula is

$$f_0 = (m + \delta) f_s / N \tag{7}$$

The above analysis can lead to a graph of the noise signal (time domain) and spectral analysis about Z(t) as shown in Figure. 1 and Figure. 2.

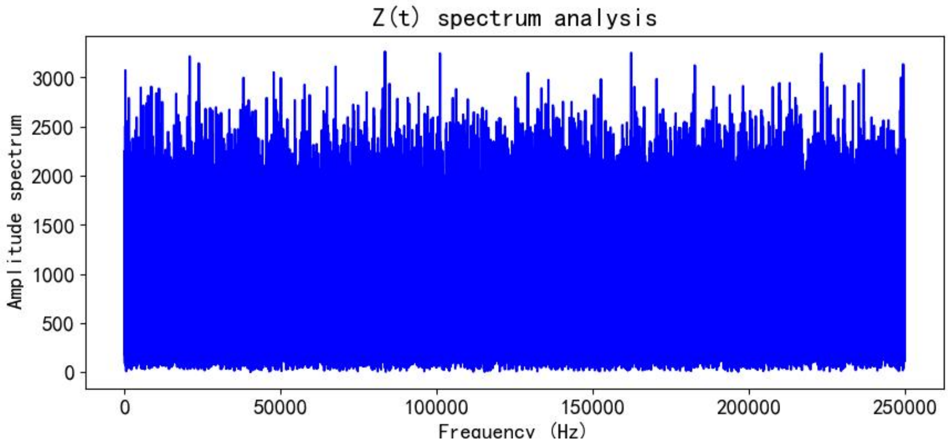

Figure 1 Spectral analysis of Z(t)

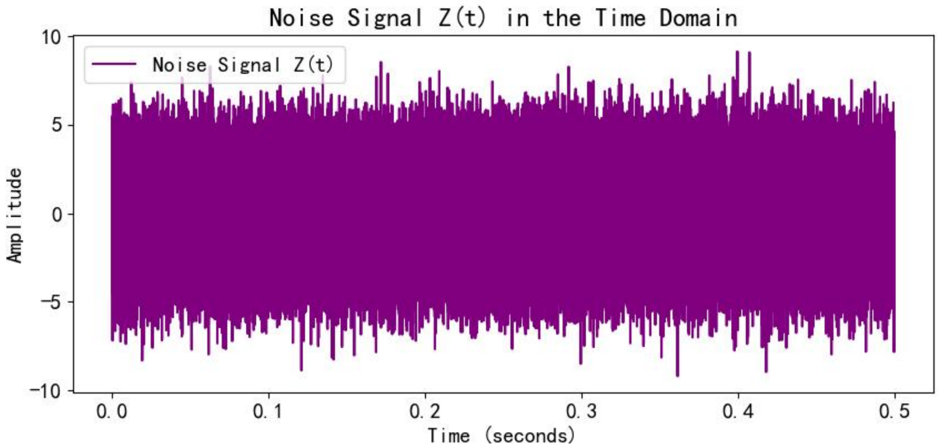

Figure 2 Noise Signal of Z(t) (Time Domain)

In order to obtain the noise characteristics, descriptive statistical analysis is used in this paper to evaluate the distributional characteristics of the noise by calculating the mean, variance and standard deviation respectively.

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{8}$$

where μ denotes the mean, n is the number of data, and x_i is the first data value

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)^2 \tag{9}$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)^2} \tag{10}$$

where σ denotes the overall variance, N is the number of overall data, x_i is the first data value, μ is the overall mean.

Finally, the mean estimate of z(t) was calculated to be 0.0058; the variance estimate of z(t) was 3.9819; and the standard deviation estimate of Z(t) was 1.9955.

Then the Q-Q plot is used to further verify the normality, if the points on the Q-Q plot of z(t) coincide with the reference line, it indicates that the distribution may obey the normal distribution. As shown in Figure 3, the paper can see that the blue points are almost completely distributed along the red line, which indicates that the distribution is very close to the normal distribution.

Points on the Q-Q graph =($Q_N(P_i), Q_X(P_i)$), where $P_i = \frac{i}{n+1}, i = 1,2, \ldots.., n$, is the sample quantile, $Q_X(P_i)$ is the normal distribution quantile.

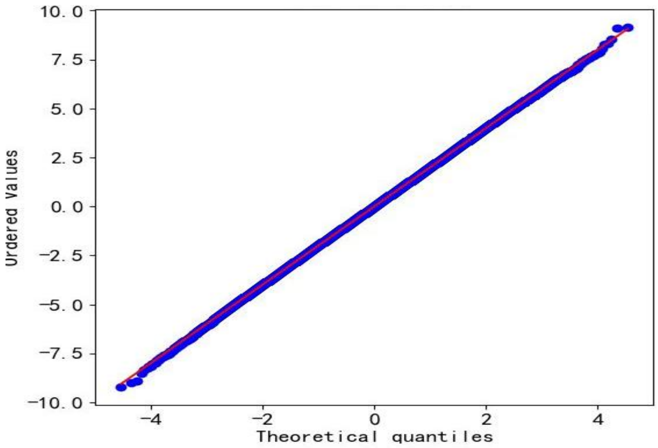

Figure 3 Q-Q Diagram for z(t)

Autocorrelation analysis is an important tool to assist in determining white noise, and for Gaussian white noise, the correlation coefficient $r_X(\tau)$ is defined as

$$r_X(\tau) = \frac{R_X(\tau)-\mu_X^2}{\sigma_X^2} \tag{11}$$

where μ_X is the mean and σ_X^2 is the variance. The autocorrelation is shown in Figure. 4, where the autocorrelation coefficient of z(t) is 1 (perfectly correlated) at lag 0, and rapidly decays to close to 0 at the other lags.This is a typical white noise autocorrelation feature, which indicates that the noise component of the signal is random and time uncorrelated.

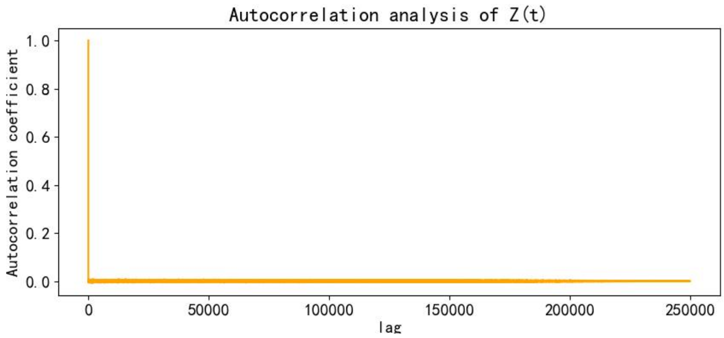

Figure 4 Correlation Plot of Noise over Time

Using probability distribution analysis models, especially the normal distribution model (Gaussian Model), as shown in Figure 5, the final judgment is Gaussian noise.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{12}$$

where μ is the mean of the distribution, σ is the standard deviation of the distribution, and π is the circumference of the circle.

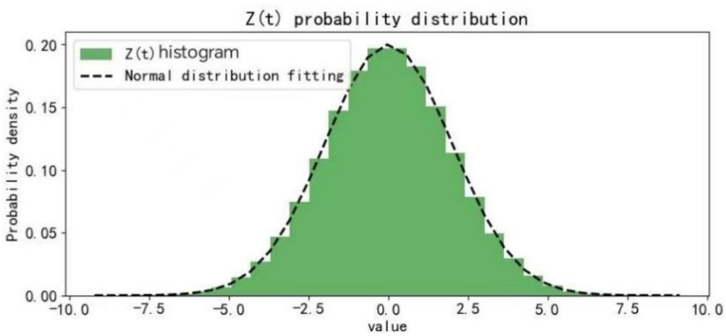

Figure 5 Fitted Plot of Z(t) Normal Distribution

A quantitative measure of the difference between a sample distribution and a reference distribution (e.g., normal distribution) using the K-S test statistic. It is calculated using the formula:

$$D = \sup_{x} \left| F_{empirical}(x) - F_{theoretical}(x) \right| \tag{13}$$

where $F_{empirical}(x)$ is the empirical distribution function of the sample, $F_{theoretical}(x)$ is the Cumulative Distribution Function (CDF) of the reference distribution, and $\sup_{x}$ denotes the maximization of the value for all x. Finally, the Kolmogorov-Smirnov test statistic was found to be 0.0015 with a p-value of 0.6575, and since p>0.05, the null hypothesis could not be rejected, and it was concluded that the sample distribution could be approximated as normal.

In summary, the fast Fourier transform (FFT) algorithm is used to avoid the complex modulo operation of signal amplitude spectrum and the complex division operation of Quinn's algorithm, which greatly reduces the amount of computation and is easy to be implemented in hardware, and the descriptive statistical analysis is used to calculate the mean estimate of z(t) to be 0.0058, the variance estimate of z(t) to be 3.9819, and the standard deviation estimate of Z(t) to be 1.9955, respectively. 1.9955. Then the white noise is judged to be Gaussian white noise by using Q-Q plots for normality verification and autocorrelation analysis to assist in judging the white noise, so as to better further analyze the noise characteristics of flight cycle one.

3 NOISE FILTERING BASED ON BAND-PASS FILTERING

In this paper, the received signal data is first read from the flight cycle and the average value of the signal is calculated, which is subtracted from the original signal to remove the 0Hz component (DC component) to ensure that the spectrum is focused on the effective AC component, thus avoiding interference at 0Hz. First a bandpass filter can be constructed by cascading a low-pass filter and a high-pass filter. Such filters allow signals in a specific frequency range to pass while blocking or attenuating signals at lower and higher frequencies[6], setting the upper and lower limits of the bandpass filters to ±20% of the initial estimated frequency (i.e., 0.8 × the initial estimated frequency and 1.2 × the initial estimated frequency), and retaining only signal components near the dominant frequency. The results are shown in Figure. 6, where the high-pass and low-pass partial cutoff frequencies can be obtained separately.

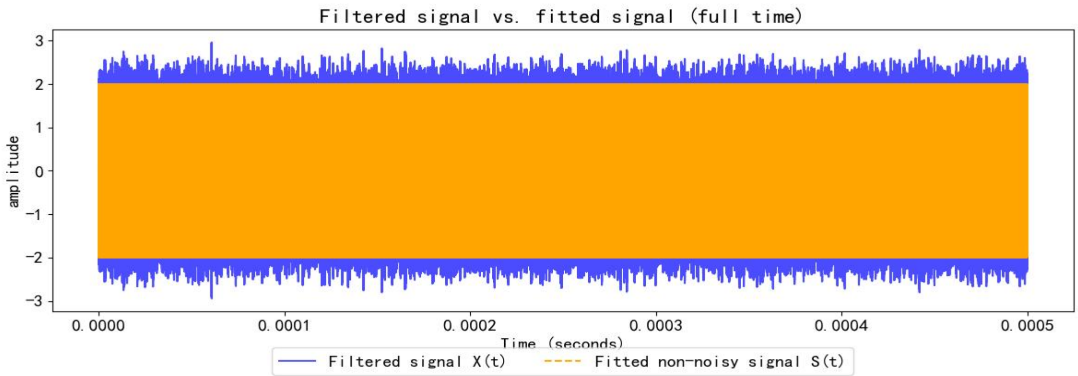

Figure 6 Filtered Signal vs. Fitted Signal (Full Time Period)

When analyzing signals to take spectral data, the most commonly used method is the Fourier transform, of which the most mature is the fast Fourier transform. There is a sampled signal of length N , where n = 0, 1, ..., N.1 refers to the time domain sampling point number. Then the signal is segmented, i.e., divided into frames, the original sampled signal is set to $X_m(n)$, the same n = 0, 1, ..., N-1, where n is the frame synchronization of the time serial number, m is the frame of the serial number, N refers to the number of sampling points in a frame. Fourier transform (DTFT) of the signal $X_m(n)$:

$$X(m, e^{jw}) = \sum_{n=0}^{N-1} w_m(n) \cdot X_m(n) \cdot e^{-jwn} \tag{14}$$

To be able to perform better discrete calculations, the discrete Fourier transform (DFT) by $w_m(n) \cdot x_m(n)$ is shown in Eq:

$$X(m, k) = \sum_{n=0}^{N-1} w_m(n) \cdot x_m(n) \cdot e^{-\frac{j2\pi nk}{2N}}, k = 0,1 \ldots\ldots, N-1 \tag{15}$$

where $|X(m,k)|$ refers to the short-time magnitude spectrum estimate of $X_m(n)$. where m refers to the time variable and k refers to the frequency variable, so $|X(m,k)|$ represents the spectrum of the dynamics. To improve the computation rate of $|X(m, I)|$, a fast Fourier transform algorithm, i.e., FFT algorithm, is used [7][8]. The resulting spectral analysis of the received signal after bandpass filtering is shown in Figure. 7, where the green curve is the spectral amplitude of the filtered signal, showing that there is only one significant peak of the dominant frequency in the spectrum. The red dashed line marks the position of the dominant frequency (frequency 40.999836 MHz), which corresponds to the dominant frequency retained after bandpass filtering. The estimated value of the dominant frequency (about 40MHz) agrees well with the theoretical value, further verifying the accuracy of the filtering and signal analysis.

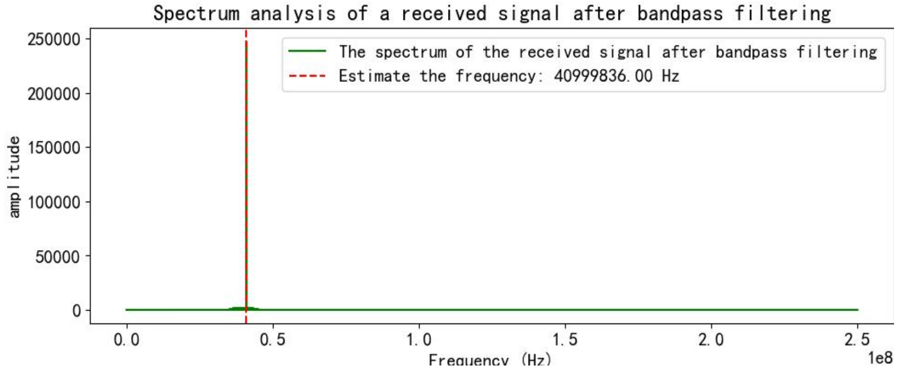

Figure 7 Spectral Analysis of the Received Signal after Bandpass Filtering

$$R(t)= X_{filtered}(t) - S_{fitted(t)} \tag{16}$$

Where $X_{filtered}(t)$ is the signal after band-pass filtering and $S_{fitted(t)}$ is the non-noise signal generated by the main frequency fitting, the residual mean is 1.1102×10^{-5} and the residual standard deviation is 4.8338×10^{-1}, so that the residual's mean is approximated to be 0, which indicates that the main signal after filtering has been well fitted. Finally, after Eq. 14 and Eq. 15, the time-domain characteristic diagram of the residual signal R(t) is shown in Figure. 8 and the spectrogram of the residual signal R(t) is shown in Figure 9.

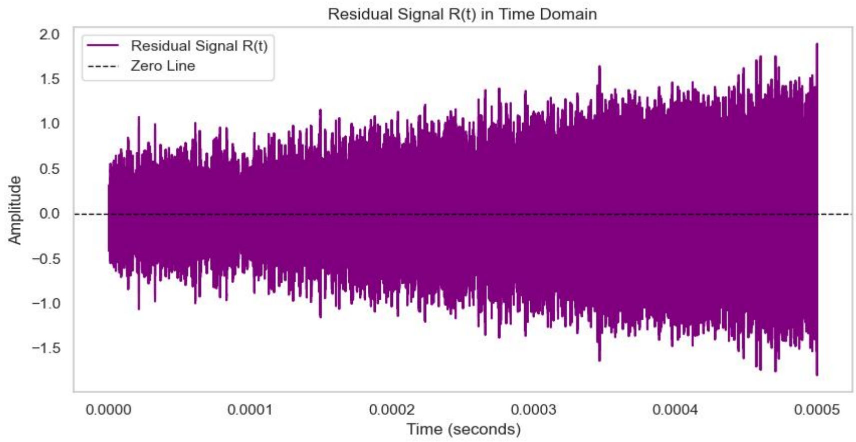

Figure 8 Time-domain Characteristics of the Residual Signal R(t)

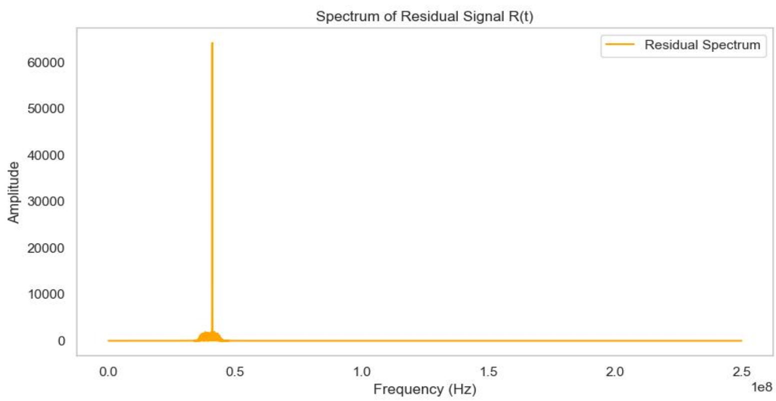

Figure 9 Spectrogram of the Residual Signal R(t)

Finally, the estimated main frequency is used to generate a pure sinusoidal signal $s(t) = Asin(2\pi ft + \varphi)$, where A is the known amplitude, f is the estimated main frequency after bandpass filtering, and φ is the known phase. By comparing the filtered signal $X_{filtered}(t)$ with the fitted non-noise signal $s(t)$, the results are shown in Figure. 10, which can verify the reasonableness and estimation accuracy of the main frequency.

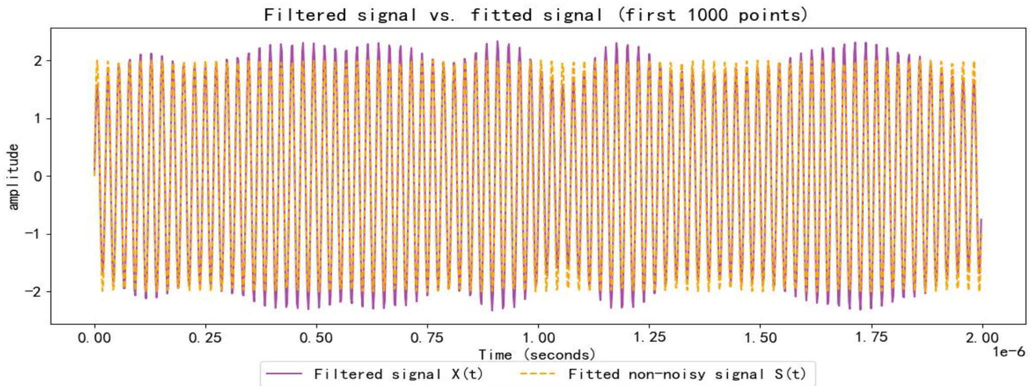

Figure 10 Plot of the Filtered Signal and the Fitted Non-Noise Signal

In summary, firstly, the band-pass filter is used to retain the components of the signal close to the frequency and exclude the noise at other frequencies to obtain the filtered signal $X_{filtered}(t)$. Then the FFT algorithm is performed again on the filtered signal $X_{filtered}(t)$ to calculate its spectrum and amplitude spectrum to obtain the spectrum analysis graph of the received signal after bandpass filtering. Next, the frequency with the largest amplitude is found in the filtered spectrum, which is used as the final estimate of the principal frequency (i.e., the frequency of the non-noise portion of the signal) Since the band-pass filtering has removed most of the noise components, this estimate will be more accurate than the preliminary frequency. The estimated main frequency is then used to generate a pure sinusoidal signal $Asin(2\pi ft + \varphi)$ for residual analysis.The mean value of the calculated residuals is approximately 0, indicating that the filtered main signal has been well fitted. Finally, the reasonableness and estimation accuracy of the main frequency are verified by comparing the filtered signal $X_{filtered}(t)$ with the fitted non-noise signal.

4 CONCLUSION

In this study, this paper presents an in-depth analysis and modeling of the frequency estimation problem of flight cycle signals, designs solution methods applicable to different scenarios, and systematically and comprehensively explores the noise characteristics, signal frequency estimation, and model performance. Noise characterization: Evaluate the noise characteristics of the flight cycle to provide a basis for frequency estimation. Frequency estimation with known partial parameters: design frequency estimation algorithms adapted to changing noise backgrounds under the condition of known amplitude and phase in the flight cycle.

In this study, the frequency estimation of complex flight cycle signals is successfully realized through theoretical modeling and algorithm design, which provides a strong support for the laser velocimetry technology of aircraft. The results not only solve the frequency estimation problem under the condition of known and unknown parameters, but also explore the signal reconstruction method of intermittent sampling, which provides a theoretical reference for future signal processing in complex environments.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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