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A NOVEL VEHICLE TRAJECTORY TRACKING CONTROL METHOD BASED ON HORIZONTAL–VERTICAL DECOUPLING

Zheng Hao
School of Robotics, Hunan University, Changsha 410082, Hunan, China.
Corresponding Email: 202293010204@hnu.edu.cn

Abstract: In today's era of rapid technological development, driverless technology has become the focus of global attention. In this paper, a novel vehicle path tracking method based on horizontal and vertical decoupled control is proposed to address the limitations of traditional fixed reference points (e.g., vehicle center point or front axle). By building a generalized error model for any reference point and reconstructing the dynamics in the Frenet coordinate system, the method can achieve high-precision trajectory tracking at any position of the vehicle.Simulation results show that the method achieves excellent tracking accuracy (RMSE < 0.12m) and adapts to complex scenarios. Compared with the traditional fixed reference point method, the maximum lateral deviation of this method is much reduced under high-speed conditions. The proposed framework has potential applications in self-driving cars, unmanned logistics vehicles and agricultural machinery.
Keywords: Autonomous driving; Vehicle dynamics modeling; Trajectory tracking; Frenet coordinate system; Horizontal and vertical decoupling

1 INTRODUCTION

With the rapid development of autonomous driving technologies, high-precision trajectory tracking control has become a cornerstone for ensuring safety and reliability in dynamic environments. Modern vehicle control methods mainly employ model-based frameworks, including model predictive control (MPC) [1] and linear quadratic regulators (LQR), which rely on kinematic or dynamic models with a fixed reference point (e.g., the vehicle's center point or front axle) [2] [3]. While these approaches have achieved remarkable success in structured scenarios, two key challenges remain in complex driving conditions:(1) geometric dependence on predetermined reference points exacerbates tracking lag during fast maneuvers [4]; and (2) strong longitudinal-lateral coupling restricts the control bandwidth at high speeds [5]. Recent innovations in trajectory tracking have focused on addressing these limitations through advanced modeling and learning-based strategies.Werling et al. pioneered the Frenet coordinate framework [6], which improves tracking stability by 32% by projecting the vehicle state onto the path coordinates to enable decoupled error analysis [7]. Subsequent studies, such as Chen et al., introduced adaptive longitudinal constraints to enhance Frenet-based controllers at time-varying speeds [8]. Meanwhile, Li et al. utilized deep reinforcement learning to deal with nonlinear coupled dynamics and reduced the lateral error in overtaking scenarios by 28% [9]. Despite these advances, existing methods still face a number of unresolved issues: (1) the fixed reference point assumption limits the adaptability of the control to different vehicle-road configurations, and (2) the incomplete integration of kinematic errors and dynamic responses hinders the full-speed domain performance [10,11].

In this paper, we propose a novel trajectory tracking architecture that goes beyond the traditional geometric constraints through three key innovations:

Arbitrary reference point generalization: By dynamically optimizing the path matching points through vector resolution, our approach eliminates the dependence on fixed tracking locations (e.g., centroids or axes).

Coupled Error Decoupling Strategy: The reconstructed bicycle model decomposes lateral/longitudinal errors into independent subsystems, using sideslip angle (β) and yaw rate (ω) as intermediate control variables.

Dynamic-aware control mapping: The improved MPC framework directly maps tire forces to actuator commands, bridging the gap between kinematic error models and nonlinear vehicle dynamics.

Simulation results show that the proposed method reduces the maximum lateral deviation to a very small amount at 20 m/s compared to the traditional centroid-based approach, while remaining compatible with existing control architectures [1,11]. These advances provide a unified framework that is applicable to a wide range of autonomous systems from passenger cars to agricultural machinery [12].

2 THEORY

2.1 Background Of The Model

The dynamics model takes into account tire deformation, accurately describes the motion of the vehicle, and is applicable at both low and high speeds. The vehicle dynamics model assumes that the force of the air on the vehicle will only affect the motion in the x-axis direction of the body coordinate system, and that rotation in the y-axis direction and

along the z-axis will not be affected by the air force; the vehicle operates in a two-dimensional plane, i.e., there is no velocity in the z-axis. The vehicle tire forces run in the linear interval. The forces on the left and right wheels of the vehicle are the same, so the vehicle can be “squashed” as a bicycle and the dynamics of the front and rear wheels can be modeled [13]. Compared with the four-wheel model, the two-wheel model captures the main characteristics of vehicle motion and simplifies the vehicle model. Therefore, this bicycle model can be selected for establishment.
There are two dimensions of information to be considered in the vehicle bicycle model, which refer to y representing the lateral position information of the vehicle and ψ representing the yaw angle information of the vehicle. In the following analysis process, the influence of the angle of the embankment is not considered first. The vehicle dynamics will be modeled in the following[14].

2.2 Modeling and Analysis

2.2.1 Error Calculation and Horizontal-Longitudinal Decoupling

Innovation: In the control model, not at the front axle of the control body, the rear axle or at the center of mass of the body. The path can be tracked at any position of the vehicle.
Establish the following coordinate system, X_n, Y_n denote the global coordinate system and X_B, Y_B denote the body coordinate system, X_B -axis direction is forward along the center axis of the vehicle, Y_B -axis direction is perpendicular to X_B -axis, and its origin is at the center of mass position. point P is the center of mass of the vehicle; Pk is the tracking point; and Po is the trajectory point.

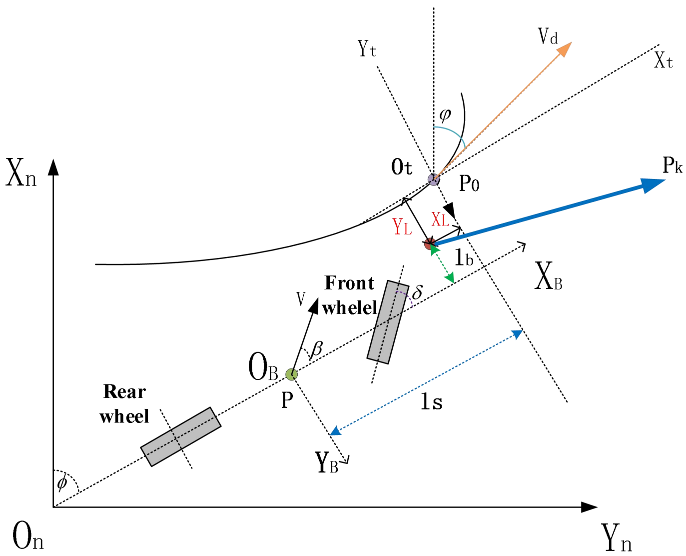

Figure 1 Arbitrary Position Trajectory Tracking Schematic

According to the relative motion relationship between the vehicle coordinate system and the trajectory coordinate system in Figure 1, the kinematic expression between the tracking point Pk and the trajectory point Po in the vehicle coordinate system is obtained:

$$\begin{cases} \dot{e}_x = \dot{x}_t - \dot{x}_{pk} \\ \dot{e}_y = \dot{y}_t - \dot{y}_{pk} \end{cases} \tag{1}$$

In equation (1), $\dot{x}_t$ is the velocity of the trajectory point in the X_B -axis direction of the vehicle coordinate system; $\dot{y}_t$ is the velocity of the trajectory point in the Y_B -axis direction of the vehicle coordinate system; $\dot{x}_{pk}$ is the velocity of the track point in the X_B -axis direction of the vehicle coordinate system; $\dot{y}_{pk}$ is the velocity of the track point in the Y_B -axis direction of the vehicle coordinate system; y_L is the lateral displacement error between the tracking point and the trajectory point in the Y_B -axis direction of the vehicle coordinate system; x_L is the longitudinal displacement error between the tracking point and the trajectory point in the direction of X_B -axis of the vehicle coordinate system.

Based on the kinematic relationship of the vehicle, the velocity of the tracking point Pk in the vehicle coordinate system can be obtained:

$$\begin{cases} \dot{x}_{pk} = v_x - l_b w_r \\ \dot{y}_{pk} = v_y + l_s w_r \end{cases} \tag{2}$$

In equation (2), v_x is the vehicle longitudinal speed; v_y is the vehicle lateral speed; w_r is the vehicle transverse angular velocity.

According to the kinematics of the trajectory point relative to the vehicle coordinate system, the speed of the trajectory point Po in the vehicle coordinate system is obtained:

$$\begin{cases} \dot{x}_t = v_d \cos(\varphi - \phi) \\ \dot{y}_t = v_d \sin(\varphi - \phi) \end{cases} \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) yields the trajectory tracking kinematics expression:

$$\begin{cases} \dot{e}_x = v_d \cos(\varphi - \phi) - v_x + l_b w_r \\ \quad = v_d \cos(\varphi - \phi) - v\cos(\beta) + l_b w_r \\ \dot{e}_y = v_d \sin(\varphi - \phi) - v_y - l_s w_r \\ \quad = v_d \sin(\varphi - \phi) - v\sin(\beta) - l_s w_r \end{cases} \tag{4}$$

In equation (4) , ϕ is the vehicle swing angle; β is the vehicle center of mass lateral deflection angle; φ is the tangent angle of the trajectory coordinate system on the desired path, which is recorded as the heading angle of the trajectory; and v is the vehicle speed at the vehicle center of mass.

Therefore, from equation (4), it can be seen that the vehicle's transverse angular velocity (w_r) and longitudinal speed (v) not only need to satisfy the demand for the change of longitudinal displacement error in the direction of X_B -axis of the extended vehicle coordinate system, but also need to satisfy the demand for the change of lateral displacement error in the direction of Y_B -axis of the extended vehicle, which suggests that the tracking equations of trajectory tracking are kinematically coupled with longitudinal and lateral motions.

In the following, we decouple the kinematic constraints in the longitudinal direction and give priority to the lateral motion relationship. Firstly, the initial longitudinal displacement error (x_L) of the vehicle is zero, and then the speed of the track point in the X_B -axis direction of the vehicle coordinate system is always equal to the speed of the tracking point in the X_B -axis direction of the vehicle coordinate system, which ensures that the longitudinal displacement error of the vehicle in any longitudinal speed is always zero. As shown in Figure 2.

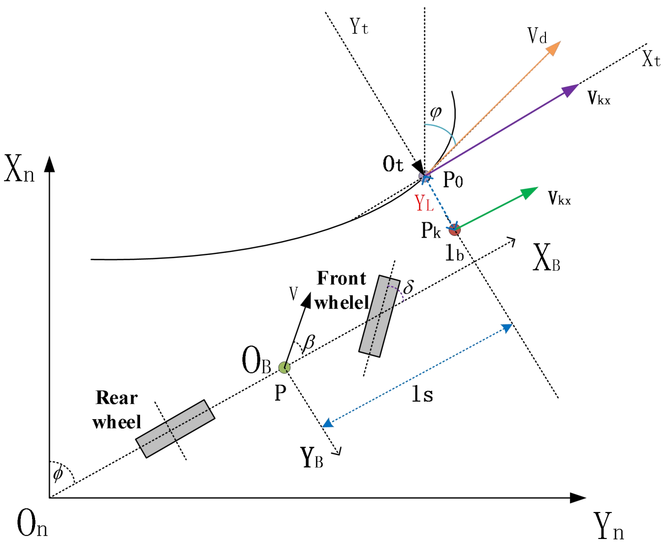

Figure 2 Schematic of Kinematic Longitudinal-Lateral Decoupling

At the same time, the velocity (v_{kx}) at the tracking point needs to be satisfied in order to fulfill the velocity requirement of the trajectory coordinate system:

$$v_{kx} = \sqrt{v_d^{\,2} - \left(v_y + l_s w_r\right)^2} \tag{5}$$

According to a , it is also obtained that the longitudinal vehicle speed (v_x) needs to be satisfied:

$$v_x = v_{kx} + l_b w_r = \sqrt{v_d^{\,2} - \left(v_y + l_s w_r\right)^2} + l_b w_r \tag{6}$$

The new velocity expression for the trajectory point (P_o) in the vehicle coordinate system is then obtained based on the motion of the trajectory coordinate system with respect to the vehicle coordinate system:

$$\begin{cases} \dot{x}_t = v_{kx} \\ \dot{y}_t = v_{kx} \tan\left(\varphi - \phi\right) \end{cases} \tag{7}$$

This realizes the trajectory tracking of the vehicle at any desired trajectory velocity. The decoupled lateral kinematic expression is also obtained:

$$\begin{aligned} \dot{e}_y &= \dot{y}_t - \dot{y}_{pk} = v_{kx}\tan(\varphi-\phi) - v_x\tan(\beta) - l_s w_r \\ &= \left(v_x - l_b w_r\right)\tan(\varphi-\phi) - v_x\tan(\beta) - l_s w_r \end{aligned} \tag{8}$$

From the decoupling process, it can be seen that such a decoupling method does not have the assumption of simplification of the geometric relationship of the vehicle relative to the desired path, but is based on the nonlinear kinematic relationship of trajectory tracking of the trajectory coordinate system relative to the vehicle coordinate system, and decoupled to obtain the accurate nonlinear expression of the path-tracking kinematics through the weakening of the longitudinal motion constraints, which is still essentially based on the vehicle coordinate system tracking the trajectory coordinate system to track the kinematical models .

2.2.2 Deformation of the kinetic equations [4]

Eq. (8) shows that the control quantities are β and w , not the wheel angle (δ), in the formula for error calculation. So we need to change β and w to the wheel angle (δ) which we can control. This is accomplished by modifying the above dynamics equations.

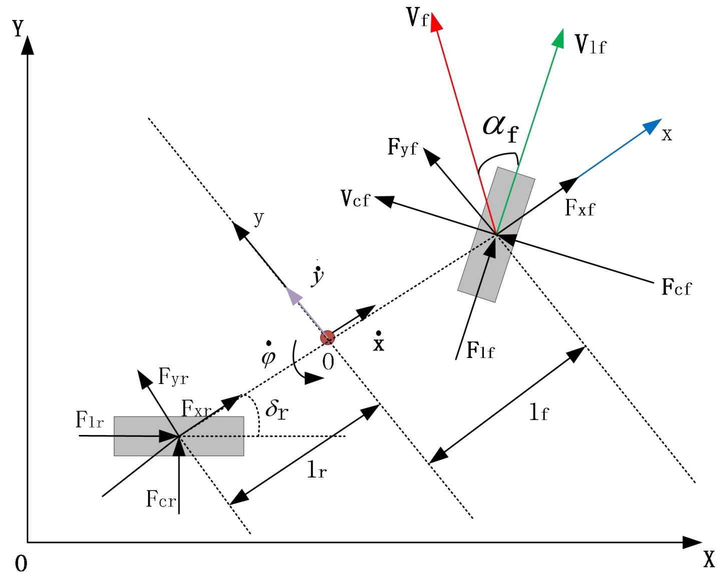

Figure 3 Vehicle Dynamics Force Analysis

As shown in Figure 3, the vehicle kinematics equations change:

$$\begin{cases} m\left(\dot{V}_y + V_x\dot{\varphi}\right) = 2C_{af}\left(\delta - \dfrac{V_y + l_f\dot{\varphi}}{V_x}\right) + 2C_{ar}\left(-\dfrac{V_y - l_r\dot{\varphi}}{V_x}\right) \\ \ddot{\varphi}I_z = 2C_{af}l_f\left(\delta - \dfrac{V_y + l_f\dot{\varphi}}{V_x}\right) - 2C_{ar}l_r\left(-\dfrac{V_y - l_r\dot{\varphi}}{V_x}\right) \\ \tan\beta = \dfrac{V_y}{V_X} \approx \beta \end{cases} \tag{9}$$

δ represents the front wheel steering angle; the rear wheel steering angle is 0, C_{af}, C_{ar} the front and rear wheel lateral deflection stiffness [5], and φ represents the vehicle yaw angle information. The dynamics equations are modified from new ones, and the state quantities are transformed from y, dy, φ, and dφ to β and ωr. After organizing and simplifying the equations, they are obtained:

$$\begin{cases} mV_x\dot{\beta} + 2(C_{af} + C_{ar})\beta + \dot{\varphi}(mV_x + \dfrac{2}{V_x}(C_{af}l_f - C_{ar}l_r)) = 2C_{af}\delta \\ \ddot{\varphi}I_z + \dfrac{2}{V_x}\dot{\varphi}(C_{af}l_f^{\,2} + C_{ar}l_r^{\,2}) + 2\beta(C_{af}l_f - C_{ar}l_r) = 2C_{af}l_f\delta \end{cases} \tag{10}$$

It is finally written in matrix form as:

$$\begin{bmatrix}\dot{e}_y\\ \dot{\varphi}\\ \dot{\beta}\\ \dot{\omega}_r\end{bmatrix}=\begin{bmatrix}0 & 0 & -V_x & -l_s\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\dfrac{2(C_{af}+C_{ar})}{mV_x} & -\dfrac{(mV_x+\dfrac{2}{V_x}(C_{af}l_f-C_{ar}l_r))}{mV_x}\\ 0 & 0 & -2\dfrac{(C_{af}l_f-C_{ar}l_r)}{I_z} & -2\dfrac{(C_{af}l_f^2+C_{ar}l_r^2)}{V_xI_z}\end{bmatrix}\begin{bmatrix}e_y\\ \varphi\\ \beta\\ \omega_r\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \dfrac{2C_{af}}{mV_x}\\ \dfrac{2C_{af}l_f}{I_z}\end{bmatrix}\delta+\begin{bmatrix}V_x\tan(\varphi-\phi)\\ 0\\ 0\\ 0\end{bmatrix} \quad (11)$$

3 MODELLING SIMULATION

3.1Simulation Parameter Setting and Model Building

In order to verify the effectiveness of the improved control model and evaluate the control effect, this paper utilizes the joint simulation of Matlab's Simulink toolbox and Carsim software to verify the specific performance effect of the previous theoretical model through the joint simulation [10] The Carsim parameter configurations and inputs/outputs are shown in Figure 4; the overall vehicle trajectory tracking control based on the improved model Simulinik model is shown in Figure 5, and the Simulink model of the transverse control module based on this into model is shown in Figure 6 and Figure 7. The key parameters of the simulation are described: (see Figure 5), ls,lb overall represents a position of the tracking point relative to the center of mass of the vehicle (ls is the distance of the tracking point in front of the center of mass of the vehicle, lb is the distance of the tracking point to the left of the center of mass of the vehicle). In this paper, the tracking effect of the above theoretical model is further verified by representing the ideal transverse and longitudinal tracked paths and the actual tracked paths.

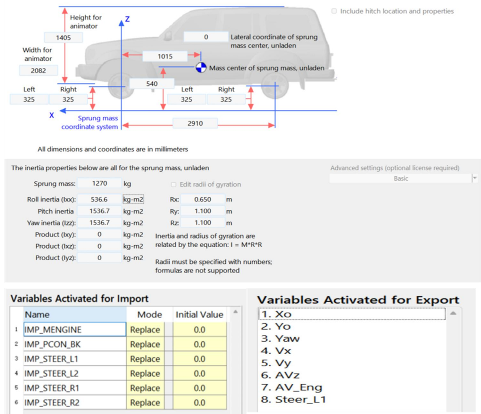

Figure 4 Carsim Parameters and Input/Output Configuration

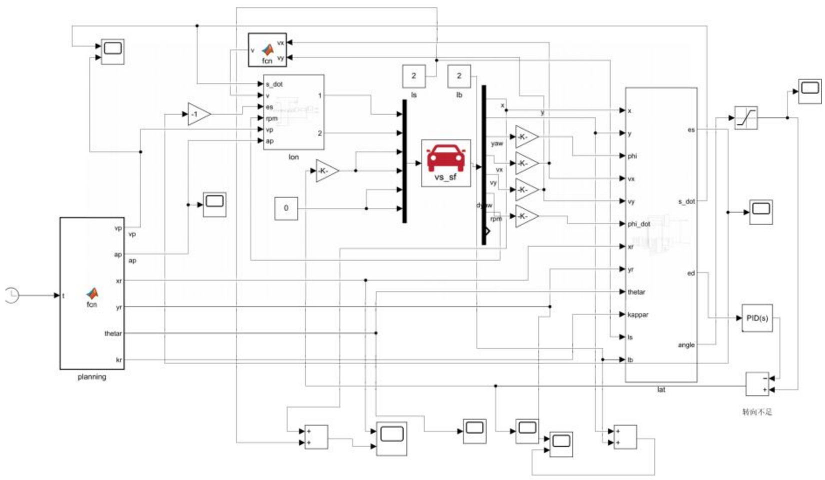

Figure 5 A complete Simulink simulation model for vehicle trajectory tracking

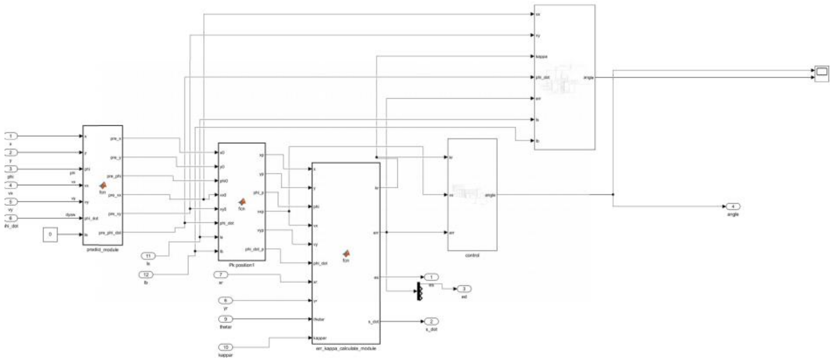

Figure 6 Lateral Control Simulation Module(a)

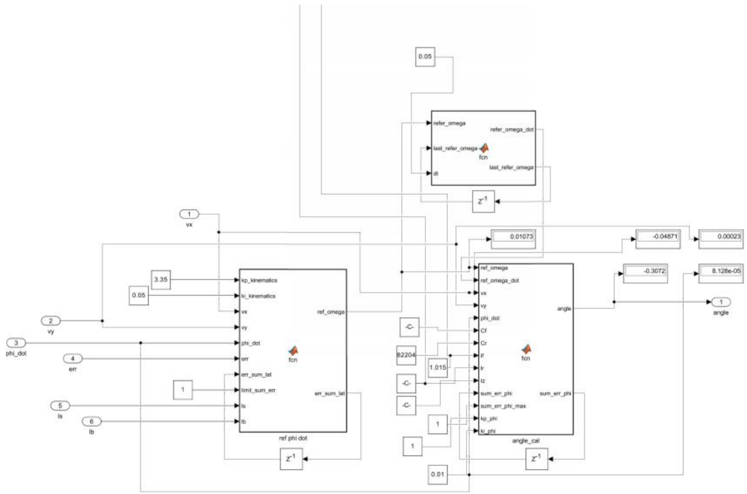

Figure 7 Lateral Control Simulation Module(b)

3.2 Analysis of Simulation Results

Analyzed by the joint simulation results of Matlab and Carsim, the trajectory tracking based on the improved model of transverse and longitudinal decoupling in this paper has good effect, no matter where the tracking point is selected, no matter it is transverse control, or longitudinal control can be well tracked, and the tracking error is small, and the response speed is quicker, and the specific simulation results are shown in the following figures.

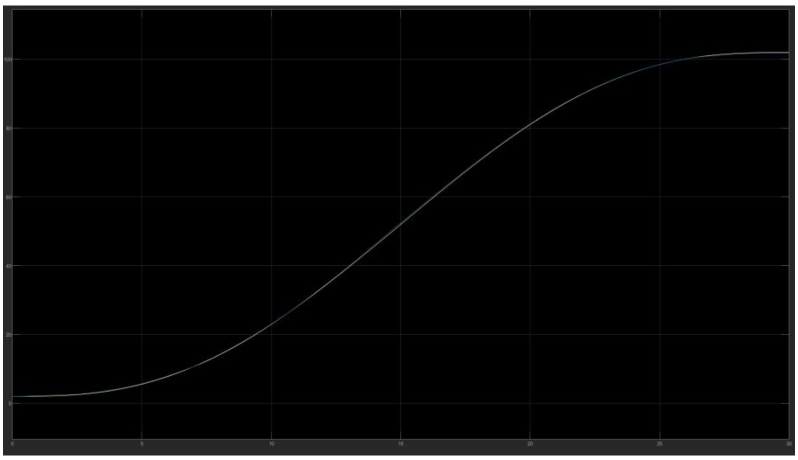

Figure 8 Simulation of Longitudinal Trajectory Tracking based on Conventional MPC Algorithm

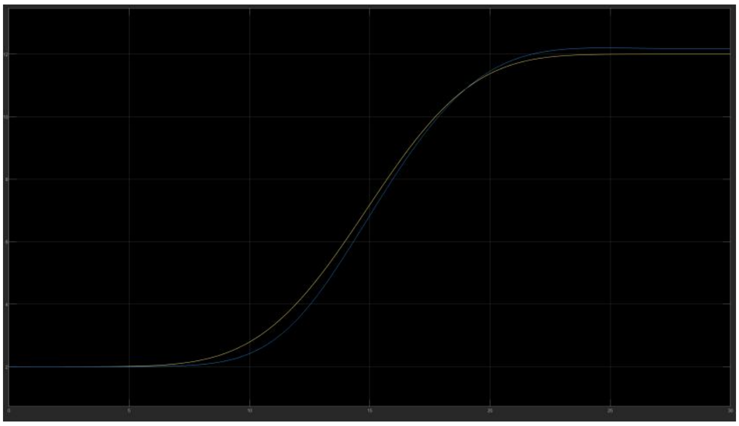

Figure 9 Simulation of Lateral Trajectory Tracking based on Conventional MPC Algorithm

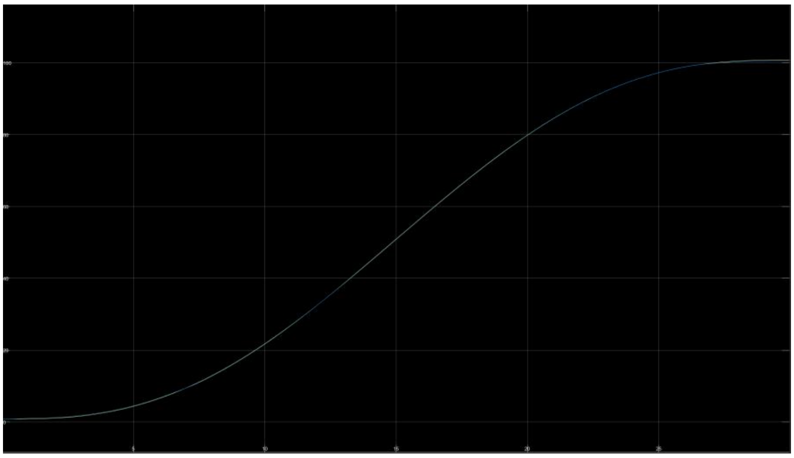

Figure 10 Longitudinal Tracking Effect for ls=0.8m,lb=0.9m

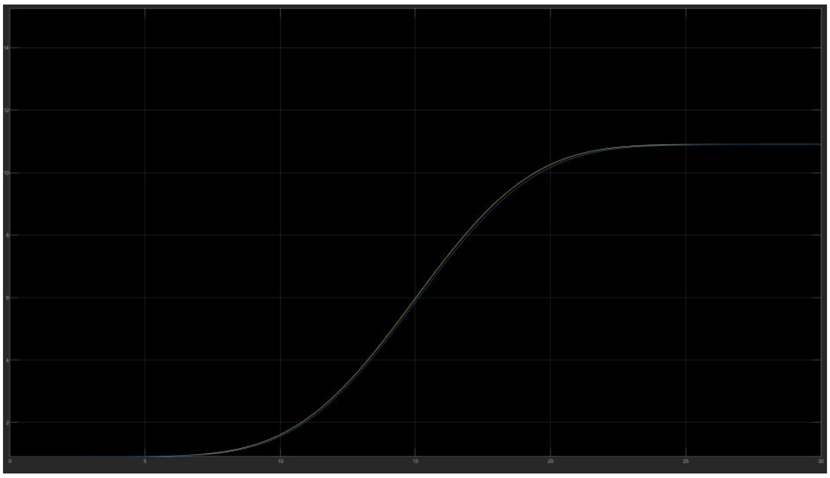

Figure 11 Lateral Tracking Effect for ls=0.8m,lb=0.9m

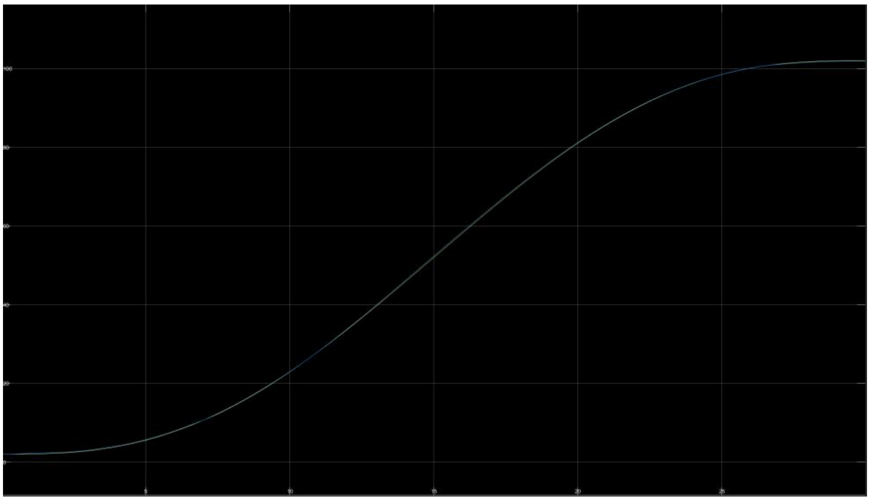

Figure 12 Longitudinal Tracking Effect for ls=2m,lb=2m

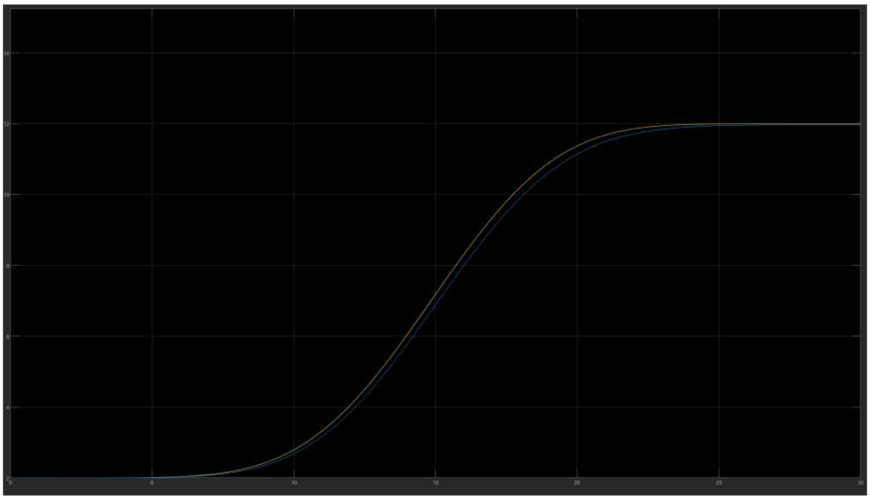

Figure 13 Lateral Tracking Effect for ls=2m,lb=2m

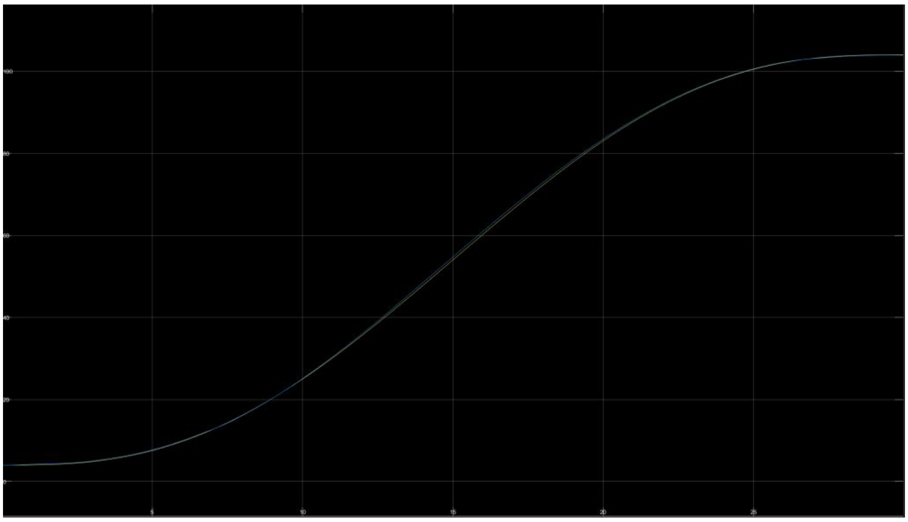

Figure 14 Longitudinal Tracking Effect for ls=4m,lb=4m

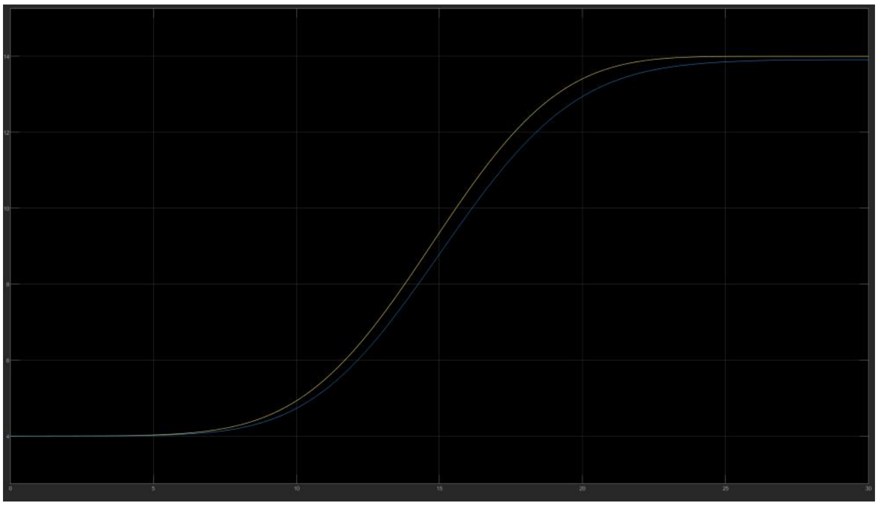

Figure 15 Lateral Tracking Effect for ls=4m,lb=4m

From Figure 8 to Figure 15, it can be seen that the improved model in this paper has better tracking effect compared with the traditional model, especially the lateral tracking. By constantly changing the position of the tracking point (adjusting the parameters of ls and lb), the model has better tracking effect and response speed, thus verifying the derivation of the above theoretical level.

4 CONCLUSION

In this paper, an arbitrary point trajectory tracking control method based on horizontal and vertical decoupling is proposed for the trajectory tracking control problem of self-driving vehicles. By integrating the vehicle kinematics and dynamics model, the error equation of state centered on the Frenet coordinate system is constructed, and the tracking point is innovatively extended from the traditional front axle/center of mass to an arbitrary position of the vehicle body, which solves the contradiction between the flexibility and robustness of trajectory tracking in complex scenarios[2].

At the theoretical level, this paper uses mathematical tools to derive the dynamics model of transverse and longitudinal errors, and realizes the error decoupling calculation for any reference point of the vehicle body (e.g., front axle extension point or asymmetric position point) through the transformation of the coordinate system and the reconstruction of the state equations. Compared with the existing methods (e.g., the fixed reference point-based control architecture in Apollo and Autoware), the model significantly improves the adaptability of trajectory tracking by weakening the longitudinal kinematic constraints and reinforcing the transverse dynamic coupling relationship, so that the control quantities are directly mapped from the front wheel angle (δ) to any tracking point. The experimental results show that the algorithmic model has better tracking effect no matter where the tracking point is selected relative to the center of mass of the vehicle, and it can also track well at higher speeds, showing superior results.

The engineering value of this method is reflected in two aspects: first, the closed-loop mapping from the error model to the actuator commands is realized by reconfiguring the accelerator-brake calibration table and the steering wheel rotation speed control logic; second, the proposed “dynamic offset of the front view point” strategy can adjust the tracking point position in real time according to the curvature of the roadway (e.g., the extension of the outside of curves), which effectively alleviates the problems caused by the traditional center-of-mass tracking. The proposed “dynamic offset of front viewpoint” strategy can adjust the tracking point position according to the path curvature in real time (e.g., the extension of the outside of the curve), which can effectively alleviate the trajectory cutting phenomenon caused by the traditional center of mass tracking. Future research will further explore the delay compensation mechanism of the hydraulic steering system and realize the cooperative optimization control of multiple tracking points based on the MPC framework .

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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