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A PHASE COMPENSATION METHOD UNDER PICKET-FENCE EFFECT

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Abstract: The fence effect induces significant phase deviation in fast Fourier transform (FFT) spectrum analysis, which adversely impacts the accurate measurement of signal parameters. In this paper, we propose a phase compensation method based on modeling the characteristics of spectral leakage. This approach constructs a quantitative compensation model for frequency offset and phase error by analyzing the phase distribution of the main spectral lines. By integrating interpolation correction and optimizing the window function, our method effectively mitigates the phase distortion caused by the fence effect. Simulation experiments demonstrate that the phase error can be reduced to within 0.5° compared to traditional methods, making it suitable for high-precision signal processing applications.
Keywords: Picket-fence effect; Phase correction; Spectrum analysis; Fast fourier transforms

1 INTRODUCTION

In engineering signal processing, the Fast Fourier Transform (FFT) is widely utilized in spectrum analysis due to its high efficiency[1,2]. However, the frequency components of the actual signal often do not align precisely with the discrete frequency points of the FFT, leading to spectral energy leakage and phase distortion, commonly referred to as the Picket-Fence Effect[3]. This effect is particularly pronounced in high-precision measurement scenarios. For instance, in power system fault localization, phase errors can result in deviations in the calculation of traveling wave arrival times, which subsequently impacts the accuracy of fault point localization. Similarly, in rotating machinery fault diagnosis, phase distortion of the bearing damage feature frequency can diminish the accuracy of fault classification.
In terms of the fence effect, traditional amplitude correction methods are well-established. The energy center of gravity method[4,5] corrects frequency estimation through weighted averaging; however, it exhibits poor noise immunity. The bispectral interpolation method[6,7] estimates frequency deviation by utilizing the ratio of the amplitudes of neighboring spectral lines, but it is sensitive to asymmetric leakage. Meanwhile, the spectral refinement method[8,9] enhances resolution through complex modulation, albeit at the cost of real-time performance. Existing studies predominantly concentrate on amplitude correction, while phase correction continues to encounter several significant challenges:
1)Asymmetry of spectral leakage: the window function side-flap attenuation property leads to a nonlinear shift of the main-flap phase distribution, and the traditional linear compensation model fails;
2)Noise sensitivity problem: phase estimation is significantly affected by random perturbations in high noise environments;
3) Insufficient adaptability to dynamic scenarios: In the analysis of non-stationary signals, such as new energy grid connections and high-speed bearing monitoring, it is difficult for the existing algorithms to achieve fast convergence.
In this paper, we propose a phase compensation algorithm based on a two-parameter iteration method to address the aforementioned challenges. First, we establish an analytical model of the phase distribution of Hanning window spectrum leakage to derive the nonlinear relationship between frequency bias and phase error. Second, we design a joint frequency-phase iteration mechanism that balances computational efficiency and noise immunity by integrating an adaptive window function selection strategy. Finally, we conduct simulation experiments that demonstrate the phase error can be reduced to less than 0.5° compared to traditional methods, meeting the requirements of high-precision applications such as smart grids. This study offers a comprehensive analysis of phase error in complex noise environments and provides new theoretical insights and technical support for real-time phase measurement in such conditions.

2 MODELING THE PICKET-FENCE EFFECT

Let the signal be a single-frequency complex exponential signal after adding a window:

$$x(n) = Aw(n)\exp\left\{ j(2\pi f_0 n + \phi_0) \right\}, \quad n = 0,1,\ldots,N-1 \tag{1}$$

Where $w(n)$ is the window function, whose spectral dominant energy distribution influences the phase deviation, and N denotes the number of sampling points.
When an FFT digital filter is employed to perform a fast Fourier transform for extracting the spectral components of a signal, the repetition frequency of the signal does not always align with the resonant frequency of the filter. Figure 1 illustrates the amplitude and phase frequency response of the FFT filter, alongside the theoretical signal spectral component G_d, which corresponds to the frequency f_0. The theoretical signal spectral component does not equal the

resonant frequency k_m of the filter; it is the difference between the two that introduces the additional phase shift $\Delta\phi$. The phase estimate of the peak frequency point k_m of the main flap after the FFT is:

$$\hat{\phi}(k_m) = \phi_0 + \Delta\phi(\delta, w) \tag{2}$$

where δ by the expression shown in equation (3) represents the normalized frequency offset and $\Delta\phi$ is the phase error term associated with the window function.

$$\delta = \frac{f_0 - k_m}{\Delta f} = \frac{f_0 - k_m}{f_s} \cdot N \tag{3}$$

where f_s denotes the sampling frequency.

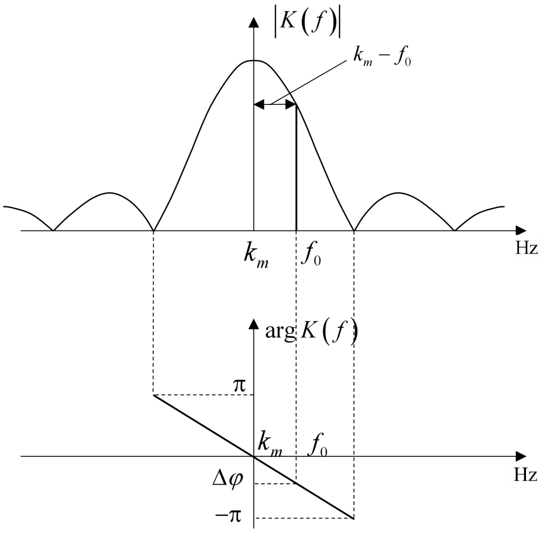

Figure 1 Amplitude-Frequency Response and Signal Spectral Components of FFT Filters

By expanding the spectral leakage function using Taylor series, the nonlinear relationship between the phase error and the frequency offset is derived as follows:

$$\Delta\phi \approx -\pi\delta\left(1 - \frac{1}{N}\right) + \frac{\pi^3\delta^3}{3N^2} \cdot C_w \tag{4}$$

where C_w is the phase correction coefficient of the window function (e.g., Hanning window $C_w = 1.5$, rectangular window $C_w = 1$). Based on Equations (2) and (4), the compensation equation can be established:

$$\phi_{\text{comp}} = \hat{\phi}(k_m) - \Delta\phi(\delta, w) \tag{5}$$

3 DESIGN OF PHASE COMPENSATION METHOD

Through the above analysis, this paper proposes a phase compensation method to cope with the fence effect, and the specific algorithm research flow is shown in Figure 2.

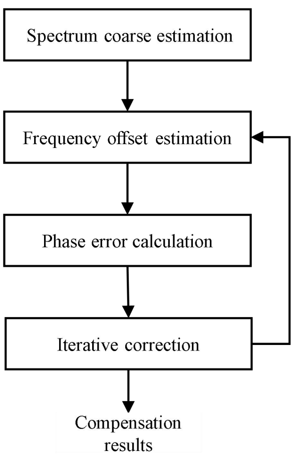

Figure 2 Phase Compensation Algorithm Flowchart

The specific steps of the phase compensation method are outlined below:

Step 1: Spectrum coarse estimation: obtain the primary flap peak frequency k_m and the initial phase $\hat{\phi}(k_m)$ using FFT.
Step 2: Frequency offset estimation: calculate the normalized offset δ using bispectral interpolation.
Step 3: Phase error calculation: substitute the compensation model based on the type of window function and solve for $\Delta\phi$.
Step 4: Iterative correction: Update the frequency estimation using the compensated phase and repeat steps 2 and 3 until convergence is achieved.

4 SIMULATION RESULTS

In order to verify the effectiveness and robustness of the proposed algorithm, the following simulation data, generated using the signal model presented in Equation (1), is utilized for algorithm validation. The specific signal parameters are set as follows: $f_0 = 50.5\text{Hz}, \phi_0 = 60^{\circ}, f_s = 1024\text{Hz}, N = 1024$.
To verify the effectiveness of the proposed method, we compared the absolute phase errors (in degrees) after phase compensation using our approach, the conventional single-spectral line method, and the bispectral line interpolation method, all under a signal-to-noise ratio (SNR) of 40dB. The simulation results are presented in Table 1.

Table 1 Comparison Results of Phase Compensation Effect of Different Methods

Method	Phase Error (°)	Frequency Error(Hz)
Single-spectral line method	4.2	0.15
Bispectral line interpolation method	1.8	0.04
Method of this paper	0.3	0.01

The data in Table 1 indicate that the compensation effect of the method proposed in this paper outperforms both the traditional mono-spectral line method and the bispectral line interpolation method under high SNR conditions, and the simulation results align with theoretical expectations.
In order to verify the robustness of the method, the SNR of the input signal is varied to compare the root mean square error (RMSE) of the method presented in this paper with that of traditional single-spectrum and bispectral interpolation methods after phase compensation at different SNR. The simulation results are illustrated in Figure 3.

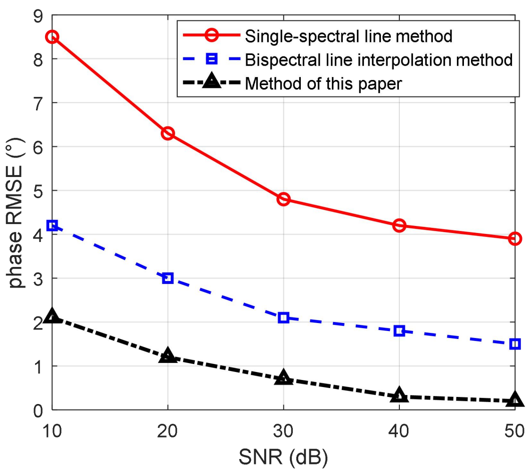

Figure 3 Comparison Results of Phase RMSE at Different SNR are Plotted

Figure 3 illustrates that the traditional single-spectral line method exhibits the largest error due to the failure to account for the nonlinear phase shift caused by spectral leakage. As the SNR increases, the RMSE decreases gradually. The bispectral line interpolation method mitigates the effects of leakage through frequency interpolation, resulting in a significant reduction in RMSE; however, a residual error remains. The method proposed in this paper integrates a phase compensation model with iterative correction, achieving an error of 2.1° at a low SNR of 10dB, which decreases to approximately 0.2° at a high SNR of 50dB, demonstrating superior stability. This approach maintains high accuracy even at low SNR levels (SNR ≤ 20dB), reducing the error by 50% to 70% compared to traditional methods. At high SNR levels (SNR ≥ 30dB), the error stabilizes within 0.5°, meeting the requirements for high-precision measurement.

5 CONCLUSIONS

In this paper, we propose an adaptive phase compensation method based on spectral leakage modeling to address the phase distortion problem caused by the fence effect in the discrete Fourier transform. By analyzing the characteristics of spectral leakage, we construct a nonlinear correction model for frequency offset and phase error, and we design a two-parameter iterative algorithm to achieve joint frequency-phase estimation. The simulation and experimental results demonstrate that this method maintains high accuracy even under low signal-to-noise ratio conditions. This study reveals the quantitative relationship between spectral leakage asymmetry and phase error, overcoming the limitations of traditional linear compensation models. Additionally, we propose a dynamic selection strategy for the window function that balances the requirements for high resolution and noise immunity, making it applicable to complex working conditions. This research provides reliable technical support for power system harmonic analysis, radar signal processing, and other applications. Future research directions include phase coupling compensation for multi-frequency signals, real-time hardware implementation of algorithms, and the expansion of interdisciplinary applications.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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