STOCK PRICE RESEARCH BASED ON ARIMA-GARCH-LSTM HYBRID MODEL

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Abstract: As financial markets become increasingly complex, the demand for stock price forecasting is growing. To capture both linear trends and volatility in sequences as well as nonlinear dependencies, this paper proposes an ARIMA-GARCH-LSTM hybrid model. First, ARIMA is used to extract linear factors, followed by GARCH to express residual volatility conditions, and finally LSTM to capture deep nonlinear features. Based on the closing prices of the Shanghai Composite Index over 1,027 trading days from 2021 to 2025, RMSE, MAE, and MAPE were used for moving forecasts and multi-indicator estimates. The experiments show that the hybrid model outperforms individual ARIMA, GARCH, or LSTM models in all metrics, confirming its accuracy and robustness. Additionally, the hybrid model demonstrates strong adaptability during periods of high volatility.

Keywords: Hybrid model; Stock price forecast; ARIMA model; GARCH family model; LSTM model

1 INTRODUCTION

The stock market is a crucial component of the modern financial market system, playing a vital role in resource allocation and reflecting economic activities. However, stock prices are influenced by various factors such as macroeconomic indicators, industrial policies, and market sentiment. Therefore, stock prices exhibit not only clear linear patterns but also volatility[1]. Moreover, there are complex nonlinear dependencies, which pose significant challenges to the accuracy of stock price predictions. Thus, effectively modeling and analyzing multiple features has become an important task both scientifically and practically.

The Autoregressive Integrated Moving Average (ARIMA) model, due to its powerful linear modeling capabilities and ease of use, is widely applied in short-term predictions of stock prices and profits. Yu Ting analyzed the adaptability of stock price series modeling based on white noise characteristics testing[2]. The study shows that some data are not suitable for direct construction of ARIMA models; using GARCH models or exponential smoothing methods can more accurately capture data features, enhancing modeling and prediction performance, thus providing a scientifically effective modeling process[3].

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is widely used to describe the volatility characteristics of financial time series, aiming for more specific descriptions of financial time series data. Liu Qi utilized the ARIMA-GARCH model to preprocess stock prices and then combined it with the CNN-BiLSTM-AT model and XGBoost algorithm to establish a hybrid prediction model[4], validating its effectiveness in achieving stock price prediction and return target goals; Xu Shuya et al. also found that joint modeling based on ARIMAARCH improves prediction stability and accuracy[5]. However, traditional GARCH models can only represent second-order matrix dynamics, with limited ability to describe higher-order nonlinear structures.

With the rise of deep learning technology, recursive neural networks (RNNs) and their derived short-term and long-term memory networks (LSTMs), due to their empirical ability to simulate long-term correlations and complex nonlinear relationships, are often used for predicting financial time series. Wang Xiaorui proposed a model that combines ARFIMA-GARCH and LSTM for disease prediction[6], which can improve feature extraction and predictive performance. The combination of GARCH and LSTM provides a model with volatility and nonlinear dynamics for disease prediction. Jiang Min et al. described the PCA-GARCH-LSTM framework, using PCA to reduce computational complexity, thereby enhancing prediction stability and accuracy[7]. Song Zhifan proposed the decomposition of Ceemdan and GARCH-LSTM methods[8], which utilize capturing multidimensional nonlinear features to significantly enhance inflation prediction performance.

This paper proposes a hybrid ARIMA-GARCH-LSTM model to address the aforementioned issues. In this method, an ARIMA model is first used to remove the linear components of time series data. Next, an appropriate GARCH model is employed to represent the temporal changes in balance[9]. Finally, the preprocessed multi-source features are input into LSTM to extract deep nonlinear relationships and predict closing prices. Empirical comparisons using historical closing values and combining sliding window prediction with multi-indicator estimation reveal that the hybrid model outperforms ARIMA, GARCH, and LSTM models in RMSE, MAE, and MAPE, demonstrating excellent predictive performance and robustness.

2 RESEARCH METHODS

In order to fully describe the linear trend, volatility clustering effect and nonlinear dependence characteristics in the

stock price sequence, this paper constructs a hybrid prediction framework based on ARIMA, GARCH family and LSTM. The specific research methods are divided into the following five parts:

2.1 Autoregressive Integrated Moving Average Model

The ARIMA (p, d, q) model eliminates the non-stationarity of the series through difference operation and fits the linear trend by using the autoregressive and moving average components.

2.1.1 Autoregressive terms

The autoregressive term represents the relationship \bar{p} between the current observation of a time series and the previous lagged observations, and can be used to capture the dependence and trend of a time series. A model with the following structure is called a p-autoregressive model, or AR (p) for short.

$$\begin{cases} x_t = \phi_0 + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \dots + \phi_p x_{t-p} + \omega_t \\ \phi_p \neq 0 \\ E(x_t) = 0, \text{Var}(\omega_t) = \delta_{\varepsilon}^2, E(\omega_t \omega_s) = 0, s \neq t \\ E(x_s \omega_t) = 0, \forall s < t \end{cases}$$
(1)

Among them, is the observation value $x_t \phi_0, \phi_1, \phi_2, \dots \phi_p$ of the current sample of the time series, represents the autoregressive coefficient, and represents the error term.

2.1.2 Differential item (I)

The primary purpose of differentiation is to ensure the stability of time series data. When the data in a time series shows trends and seasonality, these can be eliminated by differencing one or more time series data points, thereby stabilizing the time series data. The notation i(d) represents this, with the formula:

$$I(d) = \Delta x_t = x_t - x_{t-d}$$
⁽²⁾

The first difference represents the time seriesd, and the order of the difference.

2.1.3 Moving average term

This section introduces the random residual used to capture time series data. In ARIMA, it compares the current observation with past observations of the error term, helping the model correct for random fluctuations that the autoregressive part cannot capture. It is assumed that the error term is independently and identically distributed, with no autocorrelation. The moving average term is denoted as MA(q), abbreviated as MA(q):

$$\begin{cases} x_{t} = \mu + \omega_{t} + \theta_{1}\omega_{t-1} + \cdots + \omega_{t-q} \\ \theta_{q} \neq 0 \\ E(\omega_{t}) = 0, Var(\omega_{t}) = \delta_{\omega}^{2}, E(\omega_{t}\omega_{s}) = 0, s \neq t \end{cases}$$
(3)

Among them, represents the error $\bar{\omega}_r \mu_1, \mu_2, \cdots \mu_q$ term and represents the moving average coefficient. The ARIMA model is obtained by combining the above three terms, that is:

$$ARIMA(p, d, q) = AR(p) + I(d) + MA(q)$$
(4)

The basic steps are as follows: ① Stationarity test and differencing. Perform an ADF test on the original closing price series. If unstable, apply D-order differencing until the series stabilizes. ② Model identification: Combine the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the differenced series to preliminarily determine the AR order p and MA order q parameters. Parameter estimation: Use maximum likelihood estimation (MLE) or the YUIE-WAIkER equation to fit the model parameters. Model diagnostic tests: Conduct the LJUNG-BOX test on the residual series to ensure no serial correlation; ⑤ Rolling prediction: Predict the next closing price using a sliding window on the training set, obtain the residual series, and prepare it for subsequent volatility modeling and deep learning input.

2.2 Generalized Autoregressive Conditional Heteroscedasticity model

The GARCH family model is used to describe the time-varying volatility of financial time series residuals. The typical form is GARCH (p, q):

$$\begin{aligned} & \epsilon_{t} = \beta_{s}^{1/2} v_{s}, \quad v_{s} \sim i.i. d. N(0, 1), \\ & , \beta_{s} = \alpha_{0} + \sum_{i=1}^{q} \alpha_{i} \epsilon_{s-i}^{2} + \sum_{j=1}^{p} \beta_{j} \beta_{s-j} \end{aligned}$$
(5)
In wich $\alpha_{0} > 0, \ \alpha_{i} \ge 0, \ \beta_{j} \ge 0, \ \sum_{i=1}^{q} \alpha_{i} + \sum_{j=1}^{p} \beta_{j} < 1. \end{aligned}$

In this paper, GARCH(1,1) and its variants are selected for comparison in the model construction to $\{\beta_s\}$ extract the conditional volatility of the residual sequence and use it as the auxiliary input feature in the deep learning stage.

2.3 Long Short-Term Memory Network

LSTM overcomes the problem of long-term dependence $f_t i_t O_t C_t$ that is difficult to capture in traditional RNN by introducing forgetting gate, input gate, output gate and cell state transmission. The core calculation process of forgetting gate, input gate and output gate in long-term short-term memory model is shown in Figure 1: (1) forget gate:

$$f_t = \sigma(W_f[x_t, h_{t-1}] + b_f),$$
 (6)

② Input gates and candidate memories:

$$i_{t} = \sigma(W_{i}[x_{t}, h_{t-1}] + b_{i}),$$

$$\tilde{C}_{t} = \tanh(W_{C}[x_{t}, h_{t-1}] + b_{C})$$
(7)

③ Status update:

$$C_{t} = f_{t} \odot C_{t-1} + i_{t} \odot C_{t}, \tag{8}$$

④ Output gate and hidden state:

$$o_{t} = \sigma(W_{o}[x_{t}, h_{t-1}] + b_{o}),$$

$$h_{t} = o_{t} \odot \tanh(C_{t}).$$

$$\epsilon_{t} = h_{t}^{1/2} v_{t}, v_{t} \sim i.i.d. N(0,1),$$

$$h_{t} = \alpha_{0} + \sum_{i=1}^{q} \alpha_{i} \epsilon_{t-i}^{2} + \sum_{i=1}^{p} \beta_{i} h_{t-i},$$
(10)



Figure 1 LSTM Gated Information Flow Diagram

LSTM is good at learning complex nonlinear dynamic relationships from multi-dimensional inputs, and is a key tool for capturing deep patterns of stock prices in this paper.

2.4 ARIMA-GARCH-LSTM Hybrid Model

2.4.1 Steps of the combined model

(1) Data preprocessing: Fill in missing values, normalize, and differencing the original closing prices to obtain a stationary series; (2) ARIMA modeling: Fit an ARIMA model on the training set $\hat{y}_t^{ARIMA} \epsilon_t \{\epsilon_t\} \{h_t\}$ to extract linear predictions for the next time step and corresponding residuals. (3) Volatility extraction: Fit GARCH family models on the residual series to calculate conditional variance (4) LSTM training: Use the ARIMA predicted values within the time window, residual volatility, and the normalized original prices as multi-dimensional features to construct the training set, which is then input into the LSTM network for deep learning; (5) Rolling prediction and ensemble: Generate final prediction values Hybrid on the test set using the same process, and compare them with the outputs of each base model. The specific steps of ARIMA-GARCH-LSTM hybrid model are shown in Figure 2.



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2.4.2 Advantages of hybrid models

(1) The multi-source information fusion organically combines the linear trend, conditional fluctuation and nonlinear characteristics, making up for the deficiency of a single model;

(2) After ARIMA eliminates the trend, GARCH supplements the volatility, and LSTM captures the deep dynamics, which significantly reduces the prediction error;

(3) Over-fitting control uses GARCH volatility and ARIMA residual as auxiliary features to reduce the noise fitting of LSTM and improve its generalization ability;

(4) Robustness is enhanced, and the hybrid framework performs well in different market environments, especially during periods of high volatility, where reliable forecasts can be produced.

2.5 Model Evaluation Index

To evaluate the prediction performance of each model, the following indicators are adopted in this paper:

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (y_t - \hat{y}_t)^2},$$

$$MAE = \frac{1}{N} \sum_{t=1}^{N} |y_t - \hat{y}_t|,$$

$$MAPE = \frac{100\%}{N} \sum_{t=1}^{N} \left| \frac{y_t - \hat{y}_t}{y_t} \right|.$$
(11)

Among them, and are the $y_t \hat{y}_t$ Nreal value and predicted value respectively, and is the sample number. The effectiveness and superiority of the hybrid model are verified through the comprehensive comparison of the above indicators.

3 EXPERIMENT AND RESULT ANALYSIS

Based on the ARIMA-GARCH-LSTM hybrid model, the historical closing price data of Shanghai Stock Index were analyzed to verify the prediction performance of the model.

3.1 Data Selection and Description

The experimental data are from Yahoo Finance, covering 1027 trading days of Shanghai Composite Index from January 4, 2021 to April 25,2025. The data fields include: date, opening price, highest price, lowest price, closing price, adjusted opening price, and trading volume. This paper takes the closing price as the main object of analysis. In the experimental design:

(1) The closing price data of Shanghai 1027 index is divided into training set and test set in chronological order. The first 900 trading days are the training set, and the last 127 trading days are the test set.

(2) In the GARCH modeling stage, the logarithmic return series is calculated according to the closing price of the training set, and the residual is modeled.

(3) All input features (ARIMA forecast values, GARCH conditional volatility, and normalized closing prices) were standardized from minimum to maximum.

(4) In GARCH modeling, the logarithmic return is calculated using the closing price of the training set:

$$\mathbf{r}_{t} = \ln \frac{\mathbf{P}_{t}}{\mathbf{P}_{t-1}},\tag{12}$$

And fit the volatility model with its residual sequence;

Before LSTM training, Min-Max normalization is used for all features, such as normalized closing price, ARIMA prediction value and GARCH volatility:

$$x' = \frac{x - x_{\min}}{x_{\max} - x_{\min}}.$$
 (13)

3.2 Model Environment and Parameter Setting

Development environment: Python 3.8.10;

Integrated development environment: Visual Studio Code;

Key libraries: statsmodels (ARIMA), arch GARCH family models, TensorFlow, pandas, numpy, scikit-learn, matplotlib; ARIMA model: The optimal order (5,1,0) is automatically selected by AIC criterion, and MLE estimation is used on the training set;

GARCH model: based on GARCH(1,1), compared with EGARCH and APARCH, and finally took the residual volatility of GARCH(1,1) as the input;

Number of iterations: 50 epochs;

Loss function: mean square error (MSE);

Batch size: 32;

3D feature matrix: normalized price, ARIMA residual, GARCH volatility;

Data loading and preprocessing: read Excel data, process missing values, calculate logarithmic return rate, normalize

closing price;

Training/test division: the first 900 trading days for training, and the last 127 trading days for testing, training set and test set.

3.3 Analysis of Empirical Results

3.3.1 Prediction analysis of Shanghai Stock Index data set



As shown in Figure 3, the data set partitioning diagram intuitively illustrates the time series segmentation method used in the study. After normalizing the test set data of the Shanghai Composite Index, the prediction results of ARIMA, GARCH(1,1), single LSTM, and hybrid models were obtained according to the aforementioned process, and RMSE, MAE, and MAPE were calculated.



Figure 4 Log Returns and GARCH Volatility

Based on the closing price of the Shanghai Composite Index on that day, a GARCH(1,1) model was constructed using its actual logarithmic return as the dependent variable for volatility prediction. As shown in Figure 4, it can be seen that when the predicted volatility is high, the empirical logarithmic return is also high. The high consistency between these two trends indicates that the GARCH(1,1) model can capture information about yield volatility in the Shanghai Composite Index.

In order to verify the normalization effect, the fluctuations before and after the normalization of the closing price are compared. As shown in Figure 5, the gray line represents the violent fluctuations of the closing price, and the blue line represents the stable trend after normalization. The dimensional difference is eliminated by normalization, making it easier to carry out subsequent processing.



In order to comprehensively demonstrate the prediction ability of the hybrid model on the closing price of the Shanghai Composite Index, we conducted a visualization of the comparison of the hybrid model's prediction effect as shown in Figure 6. In the figure, the gray line represents the actual normalized closing price, while the blue dashed line and the green solid line respectively show the prediction results of ARIMA and LSTM.



Figure 6 Mixed Model Prediction Effect

As shown in Figure 6, the ARIMA model can only closely follow trends, but there is a deviation in price fluctuations during the mid-to-late stages of 2023; the LSTM model can track volatility locally, but it will match at the end of 2024 in extreme regions. From the comparison results, a single model has limited capability in predicting complex financial time series. Traditional statistical models are slow to respond to changes, while deep learning models are affected by noise and shocks. Therefore, extracting features from ARIMA, GARCH, and LSTM at multiple scales and modeling volatility can enhance their predictive strength and accuracy.

In order to clearly reflect the prediction performance of ARIMA-GARCH-LSTM, the data are visualized as shown in Figure 7.



Table 1 Comparison of Prediction Performance of Different Models of Shanghai Composite Index

model	RMSE	MAE	MAPE (%)
ARIMA	9.47	7.49	0.24
GARCH(1,1)	8.15	6.50	0.21
LSTM	5.78	4.63	0.15
hybrid model	4.09	3.26	0.11

As shown in Table 1, the prediction results of the hybrid model are 29.2% lower than that of the single LSTM model in RMSE, 29.6% lower than that of the single LSTM model in MAE, and 26.7% lower than that of the single LSTM model in MAPE. In order to clearly reflect the prediction effect of the hybrid model, the line chart comparing the actual closing price and the predicted value during the test period from 2021 to 2025 is shown in Figure 7.

4 CONCLUSION

In response to the limitations of traditional single models that cannot simultaneously consider linear trends, volatility clustering, and nonlinear dynamics, an ARIMA-GARCH-LSTM hybrid model is proposed. First, the ARIMA algorithm is used to extract the linear component from the stock price series. Then, the linear part is removed. Next, GARCH(1,1) is employed to characterize the time-varying volatility of resistance. Finally, LSTM is trained with the expected value from ARIMA, the conditional variance from GARCH, and normalized prices as multi-dimensional inputs to learn a deep nonlinear function. Based on empirical data from the Shanghai Composite Index covering 1027 trading days from 2021 to 2025, the RMSE, MAE, and MAPE indicators show that the proposed model significantly outperforms single models, demonstrating the rationality and stability of the model[9-11].

Compared with traditional methods, the ARIMA-GARCH-LSTM hybrid model family has the following advantages: (1)Integrate multi-source information to fully capture price dynamics;

(2)The prediction accuracy is significantly improved, and the error index decreases by two digits;

(3)Reduce the risk of overadaptation and improve the generalization ability;

(4)It has remained stable in different market environments, especially during periods of high volatility.

However, there are also shortcomings: first, the overall framework is overly complex, with extremely high demands on computer hardware resources and parameter optimization; second, the GARCH and LSTM structures along with hyperparameter tuning need further examination; third, the ability to adapt to extreme market shocks requires further investigation. Future research could focus on introducing asymmetric fat-tail distributions, combining attention mechanisms with multi-scale degradation techniques, and incorporating algorithms such as XGBoost and Transformer to build more efficient integrated frameworks.

In a word, ARIMA-GARCH-LSTM hybrid model can provide an effective way in stock prediction. It contains linear, fluctuation and non-linear characteristics in the hybrid model, which has great application possibilities and further improvement possibilities[12].

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

REFERENCES

- [1] Wang J, Zhang T, Lu T, et al. A Hybrid Forecast Model of EEMD-CNN-ILSTM for Crude Oil Futures Price. Electronics, 2023, 12(11): 2521.
- [2] Ting Y H, Long Y B, Lin D, et al. Application of a Hybrid Model Based on ICEEMDAN, Bayesian Hyperparameter Optimization GRU and the ARIMA in Nonferrous Metal Price Prediction. Cybernetics and Systems, 2023, 54(1): 27-59.
- [3] Sweeti S, Surendiran B, Dhanalakshmi R, et al. Forecasting COVID-19 Pandemic Using Prophet, ARIMA, and Hybrid Stacked LSTM-GRU Models in India. Computational and mathematical methods in medicine,2022, 1556025. DOI: https://doi.org/10.1155/2022/1556025.
- [4] Liu Qi. Stock Forecasting Based on GARCH-BiLSTM Network. Changchun University of Science and Technology, 2024. DOI: 10.26977/d.cnki.gccgc.2024.000613.
- [5] Xu Shuya, Liang Xiaoying. Research on Stock Price Prediction Based on ARIMA-GARCH Model. Journal of Henan Institute of Education (Natural Science Edition), 2019, 28(04): 20-24.
- [6] Wang Xiaorui. Application of ARFIMA-GARCH-LSTM Model in the Prediction of Hand, Foot and Mouth Disease in Shanxi Province. Shanxi Medical University, 2023. DOI: 10.27288/d.cnki.gsxyu.2023.000726.
- [7] Jiang Min, Zhang Chuyi, Sun Deshan. Stock price prediction based on PCA-GARCH-LSTM model. Software Guide, 2025, 24(01): 43-48.
- [8] Song Zhifang. Research on the Prediction of China's Inflation Rate Based on CEEMDAN-GARCH-LSTM Model. North Minzu University, 2023. DOI: 10.27754/d.cnki.gbfmz.2023.000181.
- [9] Jiang Mincong. Research on Option Trading Strategies Based on LSTM Model to Predict Volatility. University of Electronic Science and Technology of China, 2024. DOI: 10.27005/d.cnki.gdzku.2024.003958.
- [10] Hu Yamei. Stock Price Prediction Based on GARCH Family Models and LSTM Models [C]// Proceedings of the Comprehensive Innovation and Development Academic Forum, Chongqing Dingyun Culture Communication Co., Ltd. School of Statistics and Mathematics, Guangdong University of Finance and Economics; 2023, 106-109. DOI: 10.26914/c.cnkihy.2023.023020.
- [11] Yongchao J, Renfang W, Xiaodie Z, et al. Prediction of COVID-19 Data Using an ARIMA-LSTM Hybrid Forecast Model. Mathematics, 2022, 10(21): 4001.
- [12] Ma C, Wu J, Hu H, et al. Predicting Stock Prices Using Hybrid LSTM and ARIMA Model. IAENG International Journal of Applied Mathematics, 2024, 54(3): 424-432.