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ROBUST MULTI-OBJECTIVE CROP PLANNING IN MOUNTAINOUS NORTHERN CHINA VIA LP–NSGA-II–MONTE CARLO

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Abstract: This paper addresses the optimization of crop planting strategies for a mountainous rural village in northern China, characterized by diverse land resources and strict crop rotation requirements. A comprehensive mathematical modeling framework is developed, incorporating linear programming, genetic algorithms (NSGA-II), and Monte Carlo simulation to address multi-objective optimization and uncertainty in agricultural production. The study considers multiple practical constraints, including land type suitability, crop rotation, legume planting frequency, and limitations on greenhouse cultivation. The results reveal that the proposed models can effectively generate robust, adaptive cropping plans that maximize economic returns while reducing planting risks under both stable and fluctuating market conditions. The integration of Monte Carlo simulation enables the model to account for yield, price, and cost uncertainty, providing decision-makers with reliable strategies for risk management. The findings offer theoretical and practical guidance for agricultural planners and farmers, and the modeling framework has strong scalability for application in diverse environments.
Keywords: Crop planting optimization; Crop rotation; Genetic algorithm; Monte carlo simulation; Uncertainty management; Agricultural decision-making

1 INTRODUCTION

A rural village in the mountainous region of northern China possesses heterogeneous arable resources—flat dryland, terraced parcels, hillside plots, irrigated paddies, 16 conventional greenhouses and 4 smart greenhouses. Because each land class offers a distinct agro-ecological niche, crop selection must match plot characteristics while conforming to agronomic rules: legumes must appear at least once within every three-year cycle, and heavy-cropping of the same species in successive years is prohibited. To streamline field management, enhance production efficiency and curb planting risk, the present study formulates a mathematical optimisation framework that integrates land–crop suitability, expected yields, cultivation costs, market prices and projected sales volumes. The resulting model generates an optimal, rule-compliant planting strategy for every land type across the 2024 – 2030 planning horizon.
Bargués-Ribera & Gokhale studied the effect of crop rotation on disease transmission[1], and found that a reasonable crop rotation scheme can effectively reduce the accumulation of soil pathogenic microorganisms and increase crop yield. Bargués-Ribera & Gokhale studied the impact of crop rotation on disease transmission and found that a reasonable crop rotation programme can effectively reduce the accumulation of soil pathogenic microorganisms and increase crop yields. Martin studied uncertainty management at the farm level and used Monte Carlo simulation to evaluate the performance of cropping planning under market fluctuations[2], and found that the optimisation scheme considering uncertainties can significantly reduce market risks and improve the stability of agricultural returns. Zhan studied the optimisation of crop cropping structure[3], and proposed a fuzzy stochastic optimisation model, which was found to provide a more robust cropping strategy under uncertain market conditions and help to improve the science of agricultural decision-making. It is found that the model can provide a more robust planting strategy under uncertain market environment, which helps to improve the scientific nature of agricultural decision-making. Cao et al. studied the land use optimization problem based on the Non-dominated Sorting Genetic Algorithm II (NSGA-II)[4], and found that this method can effectively deal with the multi-objective optimization problem and minimize the environmental impact while maximizing the crop yield. Deb et al. proposed the NSGA-II algorithm and proved that it has high computational efficiency and reliability in solving the multi-objective optimization problems in agricultural production[5]. Chen et al. studied the application of genetic algorithms in crop planting optimisation[6], and found that the method can find suitable planting solutions according to different objectives (such as yield, cost and environmental impact) and improve the scientificity of planting decisions.
Existing research has made significant progress in crop optimisation, and methods such as linear programming, NSGA-II genetic algorithm and Monte Carlo simulation have improved the science of agricultural production decision-making to different degrees[7]. However, how to find a more efficient combination between multi-objective optimisation, uncertainty management and crop rotation pattern optimisation is still a problem that needs to be further explored in future research.

An optimal planting schedule for 2024 – 2030 is first established under relatively stable market conditions. To preserve profitability when prices fluctuate[8], the schedule is extended by introducing uncertainty parameters and re-optimising expected returns. The analysis is then broadened to capture substitution and complementarity among crops, yielding a multi-objective optimisation model that explicitly embeds these correlation effects. For a mountainous village in northern China[+], crop-planning decisions are governed by heterogeneous land categories and strict agronomic rules. The present study addresses these complexities by integrating a linear-programming model, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Monte Carlo simulation[10]. Optimal rotation plans are produced for flat dryland, terraced plots, hillside fields, irrigated paddies and other terrain classes while respecting non-recurrent cropping, legume-rotation cycles and related constraints. A staged optimisation strategy progressively enlarges the objective set—from simple return maximisation, through uncertainty management, to the inclusion of inter-crop relationships—thereby providing a systematic response to the dual challenge of enhancing income and mitigating market-driven risk.

2 METHOD

2.1 Genetic Algorithm

The genetic algorithm (GA) operates by evolving a population of candidate solutions towards better fitness according to the following selection probability:

$$p_i = \frac{f(\mathbf{x}_i)}{\sum_{j=1}^{N} f(\mathbf{x}_j)} \tag{1}$$

where p_i is the probability of selecting individual i, $f(\mathbf{x}_i)$ is its fitness, and N is the population size.
Crossover and mutation operators are applied to generate new solutions, followed by fitness evaluation and selection. The algorithm iterates until convergence or a preset number of generations is reached.
Genetic algorithm model is an optimisation algorithm based on natural selection and genetic mechanism, which provides effective solutions to problems such as search, optimisation and learning by simulating the selection, crossover and mutation processes in biological evolution. Genetic algorithms are heuristic search algorithms, and their core idea is to find the optimal or near-optimal solution to a problem by simulating biological evolution.
In a specific application, the genes of an individual can represent the acreage and plot allocation for each crop. Through the selection operation, the better individuals have a higher probability of being retained and participate in the crossover process, where the good characteristics of different individuals (e.g., higher economic efficiency or resource utilisation) are combined to generate new individuals. The mutation operation, on the other hand, ensures the diversity of the solutions and prevents the algorithm from falling into local optimal solutions. After many generations of evolution, the genetic algorithm is able to find a near-optimal planting solution to maximise the revenue.
In the current crop planting planning problem, the optimal solution involves multiple decision variables (e.g., planting area and planting location for different crops). Genetic algorithms approximate the optimal solution step by step by constructing a population containing multiple possible solutions and using a natural selection mechanism to retain those individuals (i.e., planting schemes) that perform better. Each individual in the population represents a possible solution, and through repeated selection, crossover and mutation operations, the population gradually evolves to contain more and more individuals close to the optimal solution.

2.2 Monte Carlo Simulation

The Monte Carlo simulation estimates the expected value of an output function $g(\mathbf{x})$ using repeated random sampling:

$$E[g(\mathbf{x})] \approx \frac{1}{N}\sum_{i=1}^{N} g\left(\mathbf{x}^{(i)}\right) \tag{2}$$

where N is the number of samples and $g(\mathbf{x}^{(i)})$ is the result for the ii-th random input. The standard error is given by:

$$\mathrm{SE} = \frac{\sigma}{\sqrt{N}} \tag{3}$$

where σ is the sample standard deviation.

2.2.1 Probabilistic modelling of the definitional problem

The first task of Monte Carlo simulation is to identify the randomness and uncertainty in the problem, determine the random variables, choose the probability distribution and model the relationship.

2.2.2 Generation of random samples

This is one of the core steps in Monte Carlo simulation. To simulate the randomness in the problem, random samples must be generated from a predefined probability distribution. These samples will be used as inputs to the simulation to drive the system.

2.2.3 Conduct of simulation experiments

Once sufficient random samples have been generated, simulation experiments can be conducted to examine system behaviour and estimate key statistics. Monte Carlo analysis typically entails a large number of independent runs, each using a new random sample to emulate the system under varying conditions. Multiple repetitions are performed to ensure the stability and reliability of the results. During each run, the random sample is entered into the model and the corresponding output is computed.

2.2.4Statistical analyses

Statistical analysis of a large number of experimental results enables useful information to be extracted from the simulated data. Common analytical techniques include the mean-value method, variance and standard deviation calculations, and the construction of confidence intervals.

2.2.5 Convergence and error analysis

This is the final step of the Monte Carlo simulation to ensure the reliability of the simulation results. As the number of experiments N increases, the simulation results should stabilise and converge to a realistic value. This process involves the following two aspects: convergence testing and error analysis.

2.3 NSGA-II

NSGA-II solves multi-objective optimization problems formulated as:

$$\text{Minimize:}\mathrm{F}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})\right) \tag{4}$$

$$\text{Subject to:}\mathbf{x} \in \Omega \tag{5}$$

where $\mathrm{F}(\mathrm{x})$ is the objective vector, m is the number of objectives, and Ω is the feasible set.

The algorithm uses non-dominated sorting and crowding distance to select solutions:

$$d_i = \sum\nolimits_{k=1}^{m} \frac{f_k^{i+1} - f_k^{i-1}}{f_k^{\max} - f_k^{\min}} \tag{7}$$

where d_i is the crowding distance for individual i.

Randomly generate an initial population Pt of size N, after non-dominated sorting, selection, crossover and mutation, generate the offspring population Qt, and unite the two populations to form a population Rt of size 2N; carry out rapid non-dominated sorting, and at the same time, carry out crowding computation of each individual in each non-dominated stratum, and according to non-domination and crowding, select suitable individuals to form the new parent population Pt+1; through the basic operations of genetic algorithm, generate a new offspring population Qt+1, and merge Pt+1 and Qt+1 to form the new population Rt, and repeat the above operations until the end of the procedure is satisfied. 1; generate a new offspring population Qt+1 through the basic operation of genetic algorithm, merge Pt+1 and Qt+1 to form a new population Rt, and repeat the above operations until the end of the programme is satisfied.

3 EXPERIMENTAL

3.1 Optimal Cropping Scenarios for Crops in 2024-2030

3.1.1 Data preprocessing

The data-pre-processing phase comprises three components: plot-area analysis, plot-type distribution analysis and crop-type statistics.

The size of each plot dictates the feasible cultivation scale. A census of available plots shows substantial variation: the largest parcel exceeds 200 acres, whereas several smaller plots cover less than 20 acres. To enhance overall planting efficiency, the optimisation model therefore imposes a plot-size constraint so that the planting area never surpasses a plot's maximum capacity.

Suitability for specific crops differs across plot types. Watered land favours rice or multi-season vegetables, while terraced and hillside plots support drought-tolerant cereals. A full inventory of land categories indicates that common greenhouses and terraces are most prevalent, whereas hillside and flat dryland parcels are comparatively scarce.

Crop-type statistics reveal that vegetables dominate at 43.9 % of the planted area, with cereals ranking second at 26.8 %. Although legumes occupy only a minor share, the rotation schedule still guarantees that every plot carries a legume crop at least once in a three-year cycle because of its agronomic importance.

The data are sourced from official datasets provided by the China Undergraduate Mathematical Contest in Modeling (CUMCM), the official website: https://www.mcm.edu.cn.

3.1.2 Setting of decision variables

The decision variable x_{ij}^t represents the area, measured in acres, of crop j planted on plot j in year t. Here, i denotes the plot number, ranging from 1 to 54; j indicates the crop type, ranging from 1 to 41; and t represents the year, covering a 7-year period from 2024 to 2030.

3.1.3 Restrictive condition 1

Limitations on the area of cultivated land: The total cultivated area of each plot cannot exceed its actual area.

$$\sum_{\mathrm{i}-1}^{41} x_{ij}^t \leq A_i, \forall t, i \in [1,54] \tag{8}$$

Crop growing conditions: Different types of plots can only grow suitable crops, e.g. flat dry land, terraces and hillsides can only grow food crops, and greenhouses can only grow vegetables or mushrooms.

The suitability of flat drylands, terraces and slopes for the cultivation of food crops (other than rice) in a single season each year.

$$\sum_i x_{ij}^t \leq A_j; i \in [1,26], j \in [1,16] \quad (9)$$

Rice grown in a single season per year on irrigated land.

$$\sum_i x_{ij}^t \leq A_j; i \in [27,34], j \in [1,16] \quad (10)$$

Two seasons of vegetables are grown on watered land:
Season 1: Various vegetables other than cabbage, white radish and carrot

$$\sum_i x_{ij}^t \leq A_j; i \in [27,34], j \in [17,34] \quad (11)$$

Season 2: One of cabbage, white radish and carrot

$$\sum_i x_{ij}^t \leq A_j; i \in [27,34], j \in [35,37] \quad (12)$$

Common shed:
Season 1: Various vegetables other than cabbage, white radish and carrot

$$\sum_i x_{ij}^t \leq A_j; i \in [35,50], j \in [17,34] \quad (13)$$

Season 2: Edible Mushrooms

$$\sum_i x_{ij}^t \leq A_j; i \in [35,50], j \in [38,41] \quad (14)$$

Smart Shed:
Two vegetable seasons per year (except cabbage, white radish and carrot)

$$\sum_i x_{ij}^t \leq 2A_j; i \in [51,54], j \in [17,34] \quad (15)$$

Non-cropping: The same crop cannot be grown on the same plot for two consecutive years to avoid the risk of yield reduction due to heavy cropping:

$$x_{ij}{}^t \times x_{ij}{}^{(t-1)} = 0, \forall i, j, t \quad (16)$$

Legume crop rotation requirements: Each plot should be planted with a legume crop at least once in every three years:

$$\sum_{t=T}^{T+2} \sum_{j \in [1,5] \cup [17,19]} x_{ij}^t \geq \epsilon \text{ , } i \in [1,54], T = 2024,2027 \quad (17)$$

where ϵ is a positive number that ensures a certain area of cultivation of the legume crop.
Constraints on Non-Dispersed Planting Areas: To prevent cultivation zones for each crop from becoming overly fragmented, the model imposes restrictions that confine the planting areas of the same crop to adjacent plots:

$$|x_{ij}^t - x_{i'j}^t| \leq \delta, \forall i, i' \text{is an adjacent plot of land} \quad (18)$$

where δ is the allowable acreage difference.
Limitations on the area of greenhouses: the planting area of ordinary greenhouses and intelligent greenhouses cannot exceed 0.6 acres, and the planting area of greenhouses for each crop should be limited to the specified range. For ordinary greenhouses, vegetables and edible fungi can be grown; intelligent greenhouses can only grow vegetables:

$$\sum_{j \in [17,34] \cup [38,41]} x_{ij}^t \leq 0.6 \forall t, i \in [35,50] \quad (19)$$

$$\sum_{j \in [17,34]} x_{ij}^t \leq 0.6 \forall t, i \in [51,54] \quad (20)$$

Since smart greenhouses can grow two seasons of vegetables per year, the acreage for each season needs to be constrained separately.

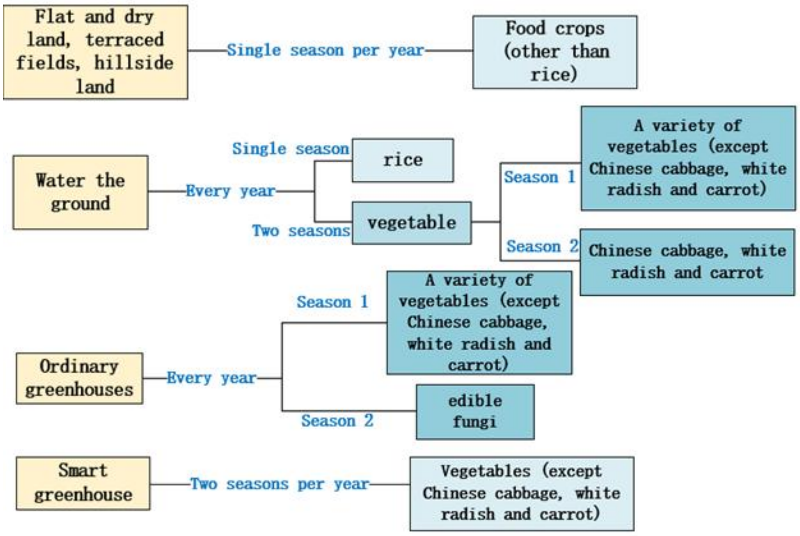

Figure 1 Crop Planting Condition Constraint Ideas

Figure 1 illustrates the core constraints for crop planting on various types of land. The figure clarifies that flat dry land, terraces, and hillsides are suitable for food crops, while greenhouses support the cultivation of vegetables and mushrooms. This visualization of constraints supports the model's practical value by ensuring that crop–land matching enhances both yield and risk mitigation.

3.1.4 Empirical results: scenario 1

Table 1 Optimal Planting Plan under Stable Market Conditions

Item	Soya bean	Black bean	......	White mushroom (Pleurotus erythrorhizonticus)	Morel mushrooms
Season 1	1	0	......	0	1
	......	......	......	......	......
	0.62	0.54	......	1	0
	0	1	......	1	1
Season 2	......	......	......	......	......
	1	0.78	......	1	1

As shown in Table 1, the allocation of different crops in ordinary and smart greenhouses is clearly presented for each season. It can be observed that soybean and black bean are arranged such that rotation requirements are satisfied, and mushrooms are predominantly scheduled in the second season for both greenhouse types. This arrangement not only maximizes the efficient use of greenhouse space but also ensures compliance with rotation constraints, helping to maintain soil health and sustain high yields.

3.1.5 Modelling and solving

This subsection assumes that if the total production of a crop exceeds its expected sales volume, the excess is no longer wasted but is sold at 50 per cent of the 2023 sales price. Therefore, the objective is to develop an optimal planting plan for the years 2024-2030 that maximises total returns under this sales rule.

3.1.6 Determine the objective function

Unlike the scenario based on stable market conditions, the portion of the crop that exceeds the expected sales volume will be sold at half price.

Therefore, the objective function needs to differentiate and treat the return on the excess portion. For a certain crop j, the return can be divided into two parts:

Firstly, production $P_j t = x_{ij} t \times Y_j$

When P_j^t is less than or equal to S_j^t:

$$Q_j^t = P_j^t \times R_j \tag{21}$$

WhenP_j^t exceeds S_j^t , the excess is sold at a 50% discounted price with the following revenue formula:

$$Q_j^t = S_j^t \times R_j + \left(P_j^t - S_j^t\right) \times R_j \times 0.5 \tag{22}$$

Thus, the total return can be expressed uniformly as:

$$Q_j^t = \min\left(P_j^t, S_j^t\right) \times R_j + \max\left(0, P_j^t - S_j^t\right) \times 0.5 \times R_j \tag{23}$$

Then, combined with the cost of cultivation, the objective function can be expressed as:

$$max \sum_{t=2024}^{2030} \sum_{i=1}^{34} \sum_{j=1}^{M} \min\left(x_i j^t \times Y_j, S_i\right) \times R_j + \max\left(0, x_i j^t \times A_j - S_i j^t\right) \times 0.5 \times R_j - x_{ij}^t \times C_j \tag{24}$$

3.1.7 Restrictive condition 2

Limitations on the area of cultivated land: The total cultivated area of each plot cannot exceed its actual area.

$$\sum_{j=1}^{41} x_{ij}^t \leq A_i, \forall t, i \in [1,54] \tag{25}$$

Crop growing conditions: Different types of plots can only grow suitable crops, e.g. flat dry land, terraces and hillsides can only grow food crops, and greenhouses can only grow vegetables or mushrooms.

$$\mathrm{x}_{\mathrm{ij}}^{\mathrm{t}} = 0 \text{ If the plot type is not suitable for growing cropsJ} \tag{26}$$

Non-cropping constraints: The same crop cannot be grown on the same plot for two consecutive years in order to avoid the risk of yield reductions associated with heavy cropping:

$$x_{ij}{}^t \times x_{ij}{}^{(t-1)} = 0, \forall i, j, t \tag{27}$$

Legume crop rotation requirements: Each plot should be planted with a legume crop at least once in every three years:

$$\sum_{t=T}^{T+2} \sum_{j \in [1,5] \cup [17,19]} x_{ij}^t \geq \varepsilon, \forall i, T = 2024{,}2027 \tag{28}$$

where ϵ is a positive number that ensures a certain area of cultivation of the legume crop.

Limitations on the area of greenhouses: the planting area of ordinary greenhouses and intelligent greenhouses cannot exceed 0.6 acres, and the planting area of greenhouses for each crop should be limited to the specified range. For ordinary greenhouses, vegetables and edible fungi can be grown; intelligent greenhouses can only grow vegetables:

$$\sum_{j \in [17,34] \cup [38,41]} \mathrm{x}_{ij}^t \leq 0.6 \forall t, i \in [35,50] \tag{29}$$

$$\sum_{j \in [17,34]} \mathrm{x}_{ij}^t \leq 0.6 \forall t, i \in [51,54] \tag{30}$$

Since smart greenhouses can grow two seasons of vegetables per year, the acreage for each season needs to be constrained separately.

Limitations on types of greenhouse crops:

Ordinary greenhouses can only grow vegetables and mushrooms, not other crops:

$$x_{ij}^t = 0, \forall j \in [1,16] \tag{31}$$

Smart greenhouses can only grow vegetables, not other crops:

$$x_{ij}^t = 0, \forall j \notin [17,37] \tag{32}$$

3.1.8 Empirical results: scenario 2

Table 2 Optimal Planting Plan with Discounted Surplus Production

Item	Soya bean	Black bean	......	White mushroom (Pleurotus erythrorhizonticus)	Morel mushrooms
	1	0	......	0	1
Season 1	......	......	......	......	......
	0.62	0.54	......	1	0
	0	1	......	1	1
Season 2	......	......	......	......	......
	1	0.78	......	1	1

Table 2 summarises the optimal crop layout when surplus output is sold at a discounted price. The plan maintains strict legume-rotation requirements and continues to schedule greenhouse capacity efficiently. Offering excess produce at a lower price encourages greater diversification, reflected in the lower dominance of any single crop within individual plots relative to the stable-market baseline.

3.2 Optimal Planting Programmes Taking into Account Uncertainty

To enhance practical relevance, the optimisation framework incorporates uncertainty in market performance, crop yields and cultivation costs. As market demand, yields and planting expenditures fluctuate over time, the model seeks a planting plan that accommodates such uncertainties while maximising expected future returns. The problem formulated under market uncertainty is inherently more complex than its stable-market counterpart; therefore, the multi-objective model is solved with the Non-Dominated Sorting Genetic Algorithm II (NSGA-II), producing a planting strategy that responds more robustly to market dynamics.

3.2.1 Uncertainty modelling: monte carlo simulation and NSGA-II.

(1) Monte Carlo simulation:

Key uncertainties include:

- Sales volume: Wheat and maize are set to grow at an annual rate of 5-10 per cent, with other crops fluctuating within a range of ±5 per cent per year.
- Yield per acre: Yield per acre may fluctuate ±10 per cent per year due to climatic influences.
- Cultivation costs: It is assumed that cultivation costs will increase by an average of 5 per cent per year.
- Selling prices: Market prices will also vary from crop to crop, with prices for vegetable crops expected to increase by 5 per cent per year, while prices for edible mushroom crops may decrease by 1-5 per cent.

Each variable can be represented by a random variable:

$$B_j^t = B_j^{2023} \times \left(1 + r_{B,j}^t\right) \tag{33}$$
$$Y_j^t = Y_j^{2023} \times \left(1 + r_{Y,j}^t\right) \tag{34}$$
$$C_j^t = C_j^{2023} \times \left(1 + r_{C,j}^t\right) \tag{35}$$
$$R_j^t = R_j^{2023} \times \left(1 + r_{R,j}^t\right) \tag{36}$$

Where $r_{sales\ volume,j}^t$ 、$r_{Y,j}^t$ 、$r_{C,j}^t$ 、$r_{R,j}^t$ are the growth rate or volatility of each variable, respectively, obeying the corresponding distribution (e.g., normal or uniform).

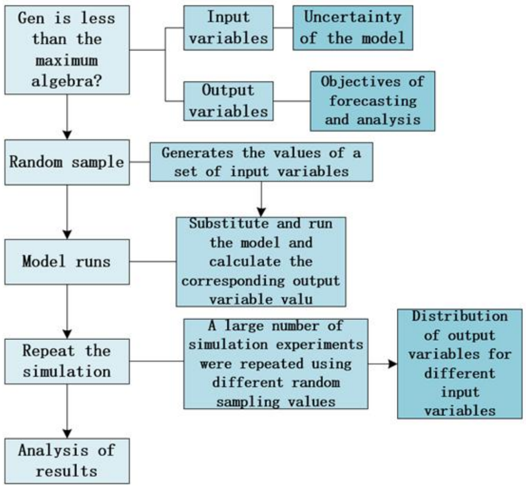

Figure 2 Monte Carlo Process

Figure 2 displays the Monte Carlo simulation process applied to crop planting strategy optimization. The flowchart demonstrates how random variables such as yield, price, and cost are incorporated into repeated simulation experiments. This process allows the model to robustly estimate outcomes under market uncertainty, providing a more reliable basis for decision-making.

(2) NSGA-II algorithm:

NSGA-II, an evolutionary technique well suited to multi-objective optimisation problems, is employed for crop-planting strategy optimisation so as to maximise expected returns while simultaneously minimising risks arising from market fluctuations.

Objective 1: Maximise returns:

$$f_1 = \sum_i \sum_j \left(P_{i,j} \cdot \min\left(C_{i,j} \cdot x_{i,j}, \mathrm{Expectedsalesvolume}\right) + 0.5 \cdot P_{i,j} \cdot \max\left(C_{i,j} \cdot x_{i,j} - \mathrm{Expectedsalesvolume}, 0\right)\right) \tag{37}$$

$P_{i,j}$: The unit sales price of the "j" crop grown on parcel "i".

$C_{i,j}$: The acreage of the "j" crop planted on parcel "i".

$x_{i,j}$: Area of the "j" th crop planted on the "i" th plot (decision variable).

Objective 2: Minimise planting costs:

$$f_2 = \sum_t \sum_j Plantingcost_{i,j} \cdot x_{i,j} \tag{38}$$

Planting cost: Unit cost of cultivation of crop "j" grown on plot "i".

3.2.2 Determine the objective function

Under uncertainty, the objective function is still to maximise expected returns over the period 2024-2030. Similar to the previous scenario based on stable market conditions, here the revenue parameters (such as sales volume, acreage, cost, and price) are treated as random variables or uncertain intervals.

When using Monte Carlo simulation, the objective function can be expressed as the average return under multiple scenarios, analogous to the stable market scenario:

$$maxE[\sum_{t=2024}^{2030}\sum_{i=1}^{34}\sum_{i=1}^{M}\left(\min\left(x_{ij}^{t}\times Y_{j}^{t},S_{j}^{t}\right)\times R_{j}^{t}-x_{ij}^{t}\times C_{j}^{t}\right] \quad (39)$$

Among them, $E[X]$ denotes the expected value over all random scenarios.
The other variables are similar.

3.2.3 Restrictive condition

Limitations on the area of cultivated land: The total cultivated area of each plot cannot exceed its actual area, viz:

$$\sum_{j=1}^{41} x_{ij}^{t} \le parcel\ area_{i}, \forall t, \forall i \quad (40)$$

Crop growing conditions: Different types of plots can only grow suitable crops, e.g. flat dry land, terraces and hillsides can only grow food crops, and greenhouses can only grow vegetables or mushrooms.

$$x_{ij}^{t} = 0_{\text{If the plot type is not suitable for growing crops}} \quad (41)$$

Non-cropping constraints: The same crop cannot be grown on the same plot for two consecutive years to avoid the risk of yield reductions associated with heavy cropping:

$$x_{ij}^{t} \times x_{ij}^{t-1} = 0, \forall i, j, t \quad (42)$$

Constraints on planting legumes once in three years: Each plot should be planted with legumes at least once in every three years:

$$\sum_{t=T}^{T+2}\sum_{j\in[1,5]\cup[17,19]} x_{ij}^{t} \ge \epsilon, \forall i, T = 2024{,}2027 \quad (43)$$

where ϵ is a positive number that ensures that the legume crop is planted on at least a certain area in three years.
Constraints on non-dispersed planting areas: The need to avoid spreading the planting areas of each crop too thinly can be addressed by adding constraints to limit the planting areas of the same crop to neighbouring plots:

$$|x_{ij}^{t} - x_{i'j}^{t}| \le \delta, \forall i, i' \text{is adjacent plots} \quad (44)$$

where δ is the allowable acreage difference. Shed Size Limit: For each shed k, it cannot exceed 0.6 acres:

$$\sum_{j\in[17,34]\cup[38,41]} x_{ij}^{t} \le 0.6 \forall t, i \in [35,50] \quad (45)$$

$$\sum_{j\in[17,34]} x_{ij}^{t} \le 0.6 \forall t, i \in [51,54] \quad (46)$$

Since smart greenhouses can grow two seasons of vegetables per year, they need to be constrained separately for each season.
Limitations on types of greenhouse crops: Ordinary greenhouses can only grow vegetables and mushrooms, not other crops. Smart greenhouses can only grow vegetables, not other crops.

$$x_{ij}^{t} = 0 \forall j \notin (1,16) \quad (47)$$

$$x_{ij}^{t} = 0 \forall j \notin (17,37) \quad (48)$$

3.2.4 Fitness function

The fitness function evaluates the benefits and costs associated with each candidate cropping plan and verifies compliance with all constraints (e.g., plot size, crop-rotation intervals, heavy-cropping prohibitions). Any violation incurs a penalty, thereby reducing the individual's fitness score.
The following are the components of the fitness function:

Calculation of proceeds (f_1)

The objective is to maximise the return, which is given by the return formula:

$$B_{i,j} = \sum_{i}\sum_{j}\left(P_{i,j}\cdot\min\left(C_{i,j}\cdot x_{i,j}, S_{i,j}\right) + 0.5\cdot P_{i,j}\cdot\max\left(C_{i,j}\cdot x_{i,j} - S_{i,j}, 0\right)\right) \quad (49)$$

Among them, $P_{i,j}$ denotes the unit selling price of the j-th crop grown on plot i; $C_{i,j}$ represents the per-acre yield of this crop; and $x_{i,j}$ indicates the cultivated area allocated to crop j on plot i;
When production exceeds expected sales, the excess may be sold at a discounted price (thus multiplied by 0.5) or may even be sold late.
Costing (f_2)
The objective is to minimise the cost of planting with the cost formula:

$$C = \sum_{i}\sum_{j} C_{i,j}\cdot x_{i,j} \quad (50)$$

Here, Planting costs i, j is the unit cost of growing the crop, multiplied by the acreage of the crop$x_{i,j}$.
Introduction of the penalty function:
Since the solution must satisfy multiple constraints (e.g., plot-size limitations, crop rotation and recropping requirements), a penalty term is introduced into the fitness function. The penalty function reduces the fitness values of individuals violating constraints, thereby guiding algorithm convergence towards constraint-satisfying solutions.
The penalty function is designed as follows: Parcel size constraint penalties:

$$penalty1 = 0 \tag{51}$$

If the sum of the acreage planted to all crops on a given plot exceeds the maximum available acreage for that plot, the penalty value is increased:

$$penalty1 \mathrel{+}= 1000 \tag{52}$$

Crop rotation requirements for legumes are penalised:

$$penalty2 = 0 \tag{53}$$

If a plot has not grown a legume crop for three years, the penalty value is increased:

$$penalty2 \mathrel{+}= 1000 \tag{54}$$

Heavy stubble restraint penalties:

$$penalty3 = 0 \tag{55}$$

If a parcel grows the same crop in two consecutive years, the penalty value is increased:

$$penalty3 \mathrel{+}= 1000 \tag{56}$$

Total adaptation value

The final fitness function contains both objective values for benefits and costs and a penalty term. The fitness function returns two objective values:

Gain (maximisation): actual gain minus the value of the penalty for violating the constraint.

Cost (minimisation): the actual cost plus the penalty value for violating the constraint.

The specific formula is:

$$fitness1 = profit - penalty1 - penalty2 - penalty3 \tag{57}$$

$$fitness2 = cost + penalty1 + penalty2 + penalty3 \tag{58}$$

Interpretation of the adaptation function

For individuals that do not satisfy the constraints, the fitness value is significantly reduced by the presence of the penalty function. This motivates the NSGA-II algorithm to prefer retaining those individuals that satisfy the constraints.

If an individual fully satisfies all constraints, the penalty value is 0 and the fitness function considers only the true values of benefits and costs.

The weight of the penalty (e.g. 1000) can be adapted to the specifics of the problem to ensure that it is large enough to affect solutions that do not satisfy the constraints.

In this way, the fitness function both drives NSGA-II to optimise towards the objective value and ensures that the solution satisfies a variety of practical constraints.

3.2.5 Empirical results: scenario 3

Table 3 Optimal Planting Plan Considering Uncertainty

Strips	Crop	Vintages	Area (of a floor, piece of land etc)	Lot 1
Lot 1	Wallabong (Bombay bean)	2024	0.68191	Lot 1
Lot 1	Crotonopsis pilosula	2025	0.68191	Lot 1
Lot 1	Wallabong (Bombay bean)	2026	0.68191	Lot 1
Lot 1	Wallabies	2027	0.68191	Lot 1
Lot 1	Wallabies	2028	0.68191	Lot 1
Lot 1	Crotonopsis pilosula	2029	0.68191	Lot 1
Lot 1	Wallabong (Bombay bean)	2030	0.68191	Lot 1
Lot 1	Millet	2024	4.99736	Lot 1
Lot 1	Millet	2025	4.99736	Lot 1
......	......	......	......	......
Lot 53	Lettuce	2029	0.6	Lot 53
Lot 53	Lettuce	2030	0.6	Lot 53
Lot 54	Lettuce	2024	0.6	Lot 54
Lot 54	Lettuce	2025	0.6	Lot 54
Lot 54	Lettuce	2026	0.6	Lot 54
Lot 54	Lettuce	2027	0.6	Lot 54
Lot 54	Lettuce	2028	0.6	Lot 54
Lot 54	Lettuce	2029	0.6	Lot 54
Lot 54	Lettuce	2030	0.6	Lot 54
Lot 53	Lettuce	2029	0.6	Lot 53

Table 3 summarizes the optimal cropping schedule for each plot over the years 2024–2030 under uncertainty. From the table, it is clear that the rotation of crops—particularly legumes—has been well planned to avoid consecutive cropping and to fulfill rotation requirements. The allocation of lettuce in adjacent plots and the alternating use of various crops across years reflects the model's ability to manage both risk and sustainability in practice.

4 CONCLUSIONS

This study develops a comprehensive optimisation model for crop planning in a mountainous village of northern China, integrating land resources, crop attributes, market demand, and the interplay of substitutability and complementarity among species. By combining linear programming, Monte Carlo simulation and the NSGA II robust-optimisation framework, the model accommodates uncertainty in yields, costs and prices, providing resilient strategies across a range of market and environmental scenarios. The inclusion of agronomic constraints such as non-recurrent planting, legume rotation cycles and strict land-type limits ensures that the recommended plans are readily implementable in real-world production.

The modelling framework is easily extendable: additional drivers, including climate variability or policy shocks, can be incorporated with minimal structural change, enabling broad application. Region-specific calibration of climate, soil and market parameters also allows tailored deployment, giving farmers locally optimised planting schedules.

Embedding the model in an agricultural information platform or mobile application converts its outputs into intuitive, visual decision tools, streamlining on-farm management for producers and agricultural officers alike. At the policy level, the same predictive engine offers early warnings of supply-demand imbalances, supports subsidy design and guides crop-structure adjustments. Coupling the optimisation core with environmental metrics advances sustainable resource use, while integration with economic analysis maximises social and financial returns. Collectively, these innovations offer robust technical and strategic support for modernising regional agriculture and promoting long-term rural resilience.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

REFERENCES

[1] Bargués-Ribera M, Gokhale CS. Eco-evolutionary agriculture: Host-pathogen dynamics in crop rotations. PLoS Computational Biology, 2020, 16(1): e1007546.

[2] Martin M. Uncertainty at the Farm–Level: Monte–Carlo Simulations for Farm Planning. Office of Economics Working Paper 2020–05–E, U.S. International Trade Commission, 2020.

[3] Zhang C. Fuzzy stochastic optimization model and method for crop planting structure. Anhui Agricultural Science, 2019, 47(18): 243-246+250.

[4] Cao K, Batty M, Huang B, et al. Spatial multi-objective land use optimization: Extensions to the non-dominated sorting genetic algorithm-II. International Journal of Geographical Information Science, 2011, 25(12): 1949-1969.

[5] Deb K, Pratap A, Agarwal S, et al. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation, 2002, 6(2): 182-197.

[6] Chen J, Zhang X, Li Y, et al. Crop planting optimization model based on genetic algorithm. Computers and Electronics in Agriculture, 2018, 150: 146-153.

[7] Ren Shuming, Xu Mengjie, Wang Qian. M&A and firm' s R&D: evidence from Chinese listed manufacturing firms. China Industrial Economics, 2017(7): 137- 155.

[8] Bowles TM, Mooshammer M, Socolar Y, et al. Long-term evidence shows that crop-rotation diversification increases agricultural resilience to adverse growing conditions in North America. One Earth, 2020, 2(3): 284-293.

[9] Mohammed A, Benkrid S, Andras P. Monte Carlo simulations for optimal crop planning under uncertain environments. Agricultural Systems, 2021, 189: 103063.

[10] Tian Y, Li X, Zhang H, et al. Agricultural production under climate change: A stochastic programming approach with scenario analysis. Journal of Agricultural Economics, 2020, 71(1): 32-49.