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A NOVEL SIMILARITY MEASURE BASED ON ENTROPY OF INTERVAL-VALUED PICTURE FUZZY SET AND ITS APPLICATION IN PATTERN RECOGNITION

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Abstract: In this paper, challenges in existing similarity measures for interval-valued picture fuzzy sets are addressed, including high computational complexity, parameter dependency, and failure to satisfy similarity axioms. First, the fundamental concepts of interval-valued picture fuzzy sets are introduced, along with definitions of distance measure, entropy measure, and similarity measure. Then, by representing interval-valued picture fuzzy sets as tetrahedrons in a 3D coordinate system and selecting 11 representative boundary points, novel entropy and similarity measures are developed based on the distances among these points. Finally, numerical examples are provided to demonstrate the effectiveness and practicality of the proposed measures, which are successfully applied to pattern recognition. Compared with existing measures, the proposed entropy and similarity measures exhibit three advantages: simplified computational procedures, reduced parameter dependency, and strict adherence to similarity axioms.
Keywords: Interval-valued picture fuzzy set; Entropy measure; Similarity measure; Pattern recognition

1 INTRODUCTION

The picture fuzzy set was originally proposed by Cuong in 2013 [1]. By introducing a “neutral membership degree” to form a triple-component structure, this framework enables describe the characterization of “hesitation”, “neutrality”, and “uncertainty” states. To address higher-order uncertainties, Cuong further generalized the three membership degrees in PFS from single-valued parameter to interval-valued representation [1], while ensuring that the sum of upper bounds for these three membership degrees remains constrained within the interval [0,1]. This generalization gives rise to interval-valued picture fuzzy set (IvPFS), which has the unique capability to model both range uncertainty in membership degrees and multi-category fuzziness types. IvPFS has been a useful tool for information processing, with applications in MCDM, pattern recognition and so on [2-4].

Similarity measure can describe the closeness between two entities. Many scholars have proposed similarity measures in FS theory and successfully applied them to pattern recognition, medical diagnosis, and other domains [5-7]. In 2018, Wei et al. developed several similarity measures for PFS [8]. These measures were successfully applied to building material recognition field, proving their effectiveness. Liu et al. proposed improved some similarity measures for IvPFS [7]. These measures were shown to be applicable to mineral recognition and strategic decision-making. Li simplified IvPFS to obtain the reduced picture fuzzy set and proposed its Dice similarity measure [9], which improved the accuracy of data processing in MCDM problems. Cao et al. devised a new similarity measure which contain the effect of refusal membership degree of IvPFS [2], and applied it in pattern recognition. In 2024, Zhao et al. integrated existing Dice similarity measures into a unified framework [10], enabling the generation of new similarity measures. Luo et al. considered the relationships among all membership functions of PFS and gave a similarity measure based on relational matrices [8]. To validate practical relevance, the authors further implemented a MCDM case study. In 2023, Kumar et al. showed the concept of IvPFM and developed similarity measures using maximum and minimum eigenvalues [11], providing a robust tool for constructing similarity measures for IvPFS using matrix-based approaches. What's more, there are also many researches about entropy and distance measures. In 2016, Wei first proposed the concept of cross-entropy for PFS [12]. In 2023, Thai et al. developed several new distance-based and entropy-based measures for PFS, exploring the application in decision support systems [13].

However, existing similarity measures of IvPFS also have some disadvantages. For instance, the similarity measures proposed by Liu and Cao are computationally complex [2, 9], making them less suitable for large-scale data processing. The parameters in the Dice similarity measure proposed by Li are significantly influenced by subjective conditions [10], which may not adequately reflect the similarity between two IvPFSs. Distance, entropy and similarity measures are three types of measures that can describe closeness, and they can be transformed into each other. In this paper, a similarity measure based on the boundary points of IvPFSs are proposed. It effectively enlarges the detailed information of membership degrees and demonstrates enhanced computational efficiency. The similarity measure shown in this paper satisfies the axiomatic criteria of similarity metrics. What’s more, in numerical case and pattern recognition, it has a better performance than the existing similarity measures for IvPFS.

2 SIMILARITY MEASURE OF IvPFS BASED ON ENTROPY

2.1 Entropy Measure of IvPFS

Entropy is an essential information measurement tool in FS theory, which can represent the degree of fuzziness of a FS. In recent years, many scholars have developed various entropy measures for types of FSs and applied them solving issues in fuzzy MCDM. In this section, a distance measure of IvPFS based on entropy measure will be constructed.

First of all, some definitions of IvPFS must be emphasized, including distance and entropy measures of IvPFS.

Definition 1: An IvPFS A on W is defined as:

$$A=\{[\varphi_{AL}(w), \varphi_{AU}(w)], [\vartheta_{AL}(w), \vartheta_{AU}(w)], [\varpi_{AL}(w), \varpi_{AU}(w)]|w\in W\} \quad (1)$$

where $[\varphi_{AL}(w), \varphi_{AU}(w)]\in D([0,1])$, $[\vartheta_{AL}(w), \vartheta_{AU}(w)]\in D([0,1])$, $[\varpi_{AL}(w), \varpi_{AU}(w)]\in D([0,1])$ are the membership, neutral membership, and non-membership degree of element w in A, satisfying: $0\le\varphi_{AU}(w)+\vartheta_{AU}(w)+\varpi_{AU}(w)\le 1$. $D([0,1])$ denotes the set of all closed subintervals in the interval $[0,1]$. The refusal membership degree of element W in A is represented as: $[\varsigma_{AL}(w),\varsigma_{AU}(w)]=\left[1-\left(\varphi_{AL}(w)+\vartheta_{AL}(w)+\varpi_{AL}(w)\right),1-\left(\varphi_{AU}(w)+\vartheta_{AU}(w)+\varpi_{AU}(w)\right)\right]$. Let $IvPFS(W)$ denote all the interval-valued picture fuzzy sets on a universe W.

Definition 2: The real function D: $IvPFS(W)\rightarrow[0,1]$ is a distance measure of IvPFS on W, if it satisfies:

(1) $0\le D(M, N)$;

(2) $D(M, N)=0$, iff $M=N$;

(3) $D(M, N)=D(N, M)$, $\forall M, N\in IvPFS(W)$

(4) $D(M, O)\ge D(M, N)$ and $D(M, O)\ge D(N, O)$, if $M\subseteq N\subseteq O$.

Definition 3: The real function E: $IvPFS(W)\rightarrow[0,1]$ is an entropy measure of IvPFS on W, if it satisfies:

(1) $E(M)=0$, iff M=([1,1], [0,0], [0,0]) or M=([0,0], [1,1], [0,0]) or M=([0,0], [0,0], [1,1]).

(2) $E(M)=1$, iff $M=\left(\left[\frac{1}{3},\frac{1}{3}\right], \left[\frac{1}{3},\frac{1}{3}\right], \left[\frac{1}{3},\frac{1}{3}\right]\right)$.

(3) $E(M)\le E(N)$, if $\forall w\in W$:

when $\varphi_{NL}\le\varpi_{NL}$ and $\varphi_{NU}\le\varpi_{NU}$: $\varphi_{ML}\le\varphi_{NL}$, $\varphi_{MU}\le\varphi_{NU}$; $\vartheta_{ML}\le\vartheta_{NL}$, $\vartheta_{MU}\le\vartheta_{NU}$; $\varpi_{ML}\ge\varpi_{NL}$, $\varpi_{MU}\ge\varpi_{NU}$. Or when $\varphi_{NL}\ge\varpi_{NL}$ and $\varphi_{NU}\ge\varpi_{NU}$: $\varphi_{ML}\ge\varphi_{NL}$, $\varphi_{MU}\ge\varphi_{NU}$; $\vartheta_{ML}\ge\vartheta_{NL}$, $\vartheta_{MU}\ge\vartheta_{NU}$; $\varpi_{ML}\le\varpi_{NL}$, $\varpi_{MU}\le\varpi_{NU}$.

(4) $E(M)\le E(M^c)$.

Let M, N be two IvPFSs. Based on the concept of arithmetic mean, the ranges of membership degree, non-membership degree, and neutral membership degree can be represented in a three-dimensional Cartesian coordinate system. The horizontal axis represents the range of membership degree $\varphi(w)$, the vertical axis represents the range of neutral membership degree $\vartheta(w)$, and the longitudinal axis represents the range of non-membership degree $\varpi(w)$. The distance measure between two IvPFSs M, N can be expressed by the average of the distance sum of several representative points in the coordinate system.

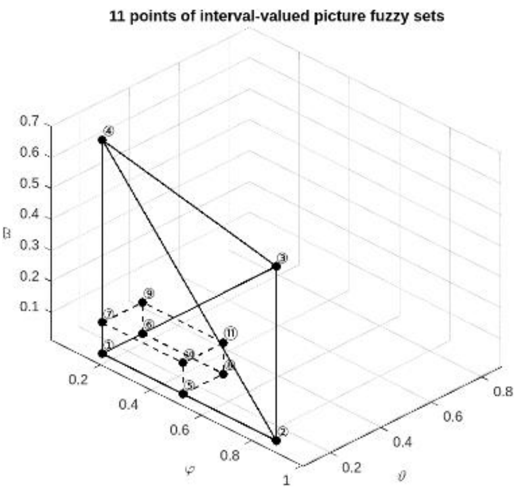

Figure 1 11Points of Interval-valued Picture Fuzzy Set

When the non-membership degree and neutral membership degree are determined as their lower bounds $\vartheta_L(w)$ and $\varpi_L(w)$, the maximum possible value of the membership degree can be calculated as either $\varphi_L(w)+\varsigma_U(w)$ or $1-\varpi_L(w)-\vartheta_L(w)$. Similarly, when the membership degree and neutral membership degree are determined as their lower bounds $\varphi_L(w)$ and $\vartheta_L(w)$, the maximum possible value of the non-membership degree can also be calculated as $\varpi_L(w)+\varsigma_U(w)$ or $1-\varphi_L(w)-\vartheta_L(w)$. Likewise, when the membership degree and non-membership degree are determined as their lower bounds $\varphi_L(w)$ and $\varpi_L(w)$, the maximum possible value of the neutral membership degree can be calculated as either $\vartheta_L(w)+\varsigma_U(w)$ or $1-\varphi_L(w)-\varpi_L(w)$. Based on these properties, a distance measure for IvPFS can be derived using the concept of arithmetic means, with representative points selected as shown in Figure 1.

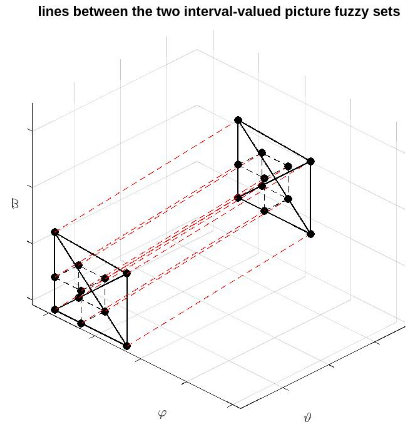

Figure 2 Lines Between the Two Interval-valued Picture Fuzzy Sets

From the relationships shown in Figure 2, the distance between two IvPFSs M and N, represented in the same coordinate system, is equal to the arithmetic mean of the distances between these representative points. Then the distance measure of IvPFS can be constructed.

Definition 4: The distance between any two IvPFSs M, N is calculated as:

$$D(M, N)(w)=\frac{1}{11}(d_1+d_2+d_3+d_4+d_5+d_6+d_7+d_8+d_9+d_{10}+d_{11}) \tag{2}$$

where $d_j(j=1,2,\cdots,11)$ represent the distance between corresponding points of two IvPFSs.

It is easy to certify that (1) satisfies the rules in Definition 2.2. Next, the entropy measure will be proposed. Considering that the IvPFS $\mathscr{M}=\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{1}{3},\frac{1}{3}\right]\right)$ represents the fuzziest case, the entropy measure is constructed based on the distance between any given IvPFS and $\mathscr{M}$. The entropy measure is as follows:

Definition 5: The entropy measure of IvPFS A is

$$E(A)=1-\frac{\sqrt{6}}{2}D(A, \mathscr{M}). \tag{3}$$

It should be noted that:

(1) When any of the membership degree, non-membership degree, or neutral membership degree of IvPFS A reaches [1,1], it has no vagueness.

(2) The IvPFS $\left(\left[\frac{1}{3},\frac{1}{3}\right], \left[\frac{1}{3},\frac{1}{3}\right], \left[\frac{1}{3},\frac{1}{3}\right]\right)$ is the fuzziest one.

(3) The closer an IvPFS is to $\left(\left[\frac{1}{3},\frac{1}{3}\right], \left[\frac{1}{3},\frac{1}{3}\right], \left[\frac{1}{3},\frac{1}{3}\right]\right)$, the fuzzier it is.

(4) An IvPFS has the same fuzziness as its complement.

In practical applications, the universe W typically contains more than one element. Therefore, when the universe consists of multiple elements, the entropy proposed in this paper is the weighted mean of the entropy of each element. What's more, it's defined that

$$E(A)=1-\frac{1}{n}\sum_{i=1}^{n}\omega_i E^i(A) \tag{4}$$

Among which $E^i(A)=1-\frac{\sqrt{6}}{2}D^i(A, M)$, ω_i represents the weight of each element $x_i\in X$, and satisfying $\sum_{i=1}^{n}\omega_i=1$.

2.2 Similarity Measure of IvPFS Based on Entropy

In this section, a new method for constructing a similarity measure of IvPFS will be introduced. By constructing a new IvPFS from two sets, the entropy of this newly constructed set is used to represent the similarity measure of these original two sets.

For two given IvPFSs $M, N\in IvPFS(W)$, the new IvPFS $N(M, N)$ is defined as follows:

$$\varphi_{N(M,N)L}(w)=\frac{1-|\varphi_{MU}(w)-\varphi_{NU}(w)|\vee|\varphi_{ML}(w)-\varphi_{NL}(w)|}{3} \tag{5}$$

$$\varphi_{N(M,N)U}(w)=\frac{1-|\varphi_{MU}(w)-\varphi_{NU}(w)|\wedge|\varphi_{ML}(w)-\varphi_{NL}(w)|}{3} \tag{6}$$

$$\vartheta_{N(M,N)L}(w)=\frac{1-|\vartheta_{MU}(w)-\vartheta_{NU}(w)|\vee|\vartheta_{ML}(w)-\vartheta_{NL}(w)|}{3} \tag{7}$$

$$\vartheta_{N(M,N)U}(w)=\frac{1-|\vartheta_{MU}(w)-\vartheta_{NU}(w)|\wedge|\vartheta_{ML}(w)-\vartheta_{NL}(w)|}{3} \tag{8}$$

$$\varpi_{N(M,N)L}(w)=\frac{1-|\varpi_{MU}(w)-\varpi_{NU}(w)|\vee|\varpi_{ML}(w)-\varpi_{NL}(w)|}{3} \tag{9}$$

$$\varpi_{N(M,N)U}(w)=\frac{1-|\varpi_{MU}(w)-\varpi_{NU}(w)|\wedge|\varpi_{ML}(w)-\varpi_{NL}(w)|}{3} \tag{10}$$

It can be concluded that $N(M, N)$ is an IvPFS on W, and satisfying $0 \le \varphi_{N(M,N)L}(w) \le \varphi_{N(M,N)U}(w) \le \frac{1}{3}$; $0 \le \vartheta_{N(M,N)L}(w) \le \vartheta_{N(M,N)U}(w) \le \frac{1}{3}$; $0 \le \varpi_{N(M,N)L}(w) \le \varpi_{N(M,N)U}(w) \le \frac{1}{3}$. In addition, there are:

$$\varsigma_{N(M,N)L}(w) = \frac{1}{3}(|\varphi_{MU}(w) - \varphi_{NU}(w)| \wedge |\varphi_{ML}(w) - \varphi_{NL}(w)| + |\vartheta_{MU}(w) - \vartheta_{NU}(w)| \wedge |\vartheta_{ML}(w) - \vartheta_{NL}(w)| + |\varpi_{MU}(w) - \varpi_{NU}(w \quad (11)$$

$$\varsigma_{N(M,N)U}(w) = \frac{1}{3}(|\varphi_{MU}(w) - \varphi_{NU}(w)| \vee |\varphi_{ML}(w) - \varphi_{NL}(w)| + |\vartheta_{MU}(w) - \vartheta_{NU}(w)| \vee |\vartheta_{ML}(w) - \vartheta_{NL}(w)| + |\varpi_{MU}(w) - \varpi_{NU}(w \quad (12)$$

so the following Theorem 1 holds.

Theorem 1: If E is the entropy measure of IvPFS, the $S(M, N) = E(N(M, N))$ is the similarity measure of IvPFS *A* and *B*.

3 RESULTS AND ANALYSIS

3.1 Evaluating the Effectiveness of Entropy Measure

In this section, two numerical examples will be shown to verify the effectiveness of our entropy measure constructed in section 2.1.

Example 1: Let M, N, O be three IvPFSs on universe W, with M=([0.1, 0.2], [0.1, 0.3], [0.4, 0.5]), N=([0.25, 0.35], [0.1, 0.2]), [0.3, 0.35]), O=([0.1, 0.3], [0.03, 0.5], [0.1, 0.2]). The entropy of three IvPFSs is calculated as follows: $E(M)=0.6345$, $E(N)=0.7451$, $E(O)=0.5469$. Hence $E(N)>E(M)>E(O)$.

This example proves that the entropy measure formula proposed in this paper is computable, then it will be proved that this entropy measure formula is effective.

Example 2: Let M, N, O be three IvPFSs on universe W, with

M=([0.04, 0.1125], [0.09, 0.25], [0.2775, 0.4375]),

N=([0.008, 0.0429], [0.027, 0.125]), [0.3859, 0.5781]),

O=([0.0016, 0.015], [0.0081, 0.0625], [0.478, 0.6836]).

From the perspective of both mathematical operations and intuition, the entropy of these three IvPFSs should satisfy the following relationship $E(M)>E(N)>E(O)$. Calculating using the proposed entropy measure formula, it obtains $E(M)=0.5292$, $E(N)=0.4758$, $E(O)=0.4000$, confirming the relationship $E(M)>E(N)>E(O)$. This indicates that the proposed entropy measure aligns with this statement. This example verifies the entropy measure formula proposed in this paper is effective.

3.2 Comparison with Existing Similarity Measures

In this section, an example will be used to verify the effectiveness of the entropy-based similarity measure proposed in this paper, and the results are compared with existing similarity calculation results.

Example 3: Let O, A, B, C be three IvPFSs on universe W, with

O=([0.05, 0.1], [0.18, 0.29], [0.43, 0.57]),

A=([0.26, 0.31], [0.12, 0.24]), [0.21, 0.39]),

B=([0.32, 0.37], [0.15, 0.28], [0.05, 0.12]),

C=([0.23, 0.46], [0.1, 0.15], [0.31, 0.36]).

Calculate the similarity between A, B, C and O respectively, the results are shown in Table 1.

Table 1 Similarity Measure of *OA*, *OB*, *OC*

SM	(O, A)	(O, B)	(O, C)	result
S	0.6074	0.6162	0.6328	C>B>A
S_{Li}	0.6857	0.8495	0.8774	C>B>A
S_{CSM}^1	0.2086	0.3121	0.3397	C>B>A
S_{CSM}^2	0.2759	0.3312	0.3799	C>B>A
S_{CsSM}^1	0.8181	0.9048	0.9178	C>B>A
S_{CsSM}^2	0.5686	0.7126	0.8044	C>B>A
S_{CsSM}^3	0.8181	0.8910	0.8763	B>C>A
S_{CsSM}^4	0.3387	0.3899	0.6129	C>B>A
S_{CtSM}^1	0.5195	0.6346	0.6569	C>B>A
S_{CtSM}^2	0.5159	0.6128	0.5914	B>C>A
S_{StSM}	0.3976	0.7337	0.8139	C>B>A
S_{GSM}	1	1	1	null
S_{DSM}^1	0.6242	0.8277	0.8859	C>B>A
S_{DSM}^2	0.6840	0.7739	0.8505	C>B>A

S_{DSM}^3	0.6242	0.8277	0.8859	C>B>A
S_{DSM}^4	0.6840	0.7739	0.8505	C>B>A

Note: S is the entropy-based similarity measure proposed in this paper, others are existing similarity measures.

According to the analysis of the calculation results in Table 1, the similarity ranking results obtained by the similarity measure proposed in this paper are consistent with the ranking results obtained by most of the existing similarities and have good applicability. Table 1 also clearly shows certain limitations of existing similarity measures. For instance, the computational results displayed by S_{CSM}^1 and S_{CSM}^2 are only approximately 0.3. It could also be argued that when the similarity measure is as low as 0.3, the two IvPFSs might be considered dissimilar. S_{GSM}^1 fails to compute the similarity value under a single criterion. S_{Li}, S_{CSM}^1, S_{CsSM}^1, S_{CsSM}^2, S_{CtSM}^1, S_{StSM}^1, S_{GSM}^1, S_{DSM}^1, S_{DSM}^3 don't consider the influence of refusal membership degree and the computational results of S_{CsSM}^1 and S_{CsSM}^3, S_{CtSM}^1 and S_{CtSM}^2 suggest that refusal membership degree affects the ranking outcomes. The similarity measure S proposed in this paper demonstrates a more robust computational outcome, which also takes into account the influence of refusal membership degree, so S is a more persuasive similarity measure.

3.3 Application in Pattern Recognition

Pattern recognition is a technical science that utilizes computers to classify, identify and analyze various types of data. By steps such as data preprocessing, feature extraction, model selection and training, classification and recognition, as well as performance evaluation, it enables the computer to automatically identify and interpret patterns with the input data. This technology has been widely applied in numerous fields including image processing, speech processing, text processing, signal processing, automated control, financial analysis, bioinformatics, security, recommendation systems, and natural language processing. It is driving the advancement of AI and providing efficient, intelligent solutions across various industries.

Let $O_1, O_2, \cdots, O_m$ be the unknown patterns, O be the unknown pattern in the feature space $\mathscr{F}=\{\mathscr{F}_1, \mathscr{F}_2, \cdots, \mathscr{F}_n\}$ and $\{\phi_1, \phi_2, \cdots, \phi_p\}$ be p attributes while the attribute values illustrate certain characteristics patterns to be considered. Through the data based on the extracted pattern characteristics and feature space analysis, the IvPFSs are constructed for each pattern as $O_1^{'}, O_2^{'}, \cdots, O_m^{'}$ respectively. Then calculate the similarity measure between each pattern $O_i^{'}$, i=1, 2, $\cdots$, m and $O^{'}$ by using any of the proposed similarity measures for IvPFS. The classification decision is made by identifying the candidate pattern demonstrating the highest similarity index relative to the target pattern. A pattern recognition algorithm is presented as follows.

Step1: Formulate IvPFS models for both reference pattern and unclassified samples.

Setp2: Calculate the similarity measure between $O_i^{'}$ and $O^{'}$.

Step3: Determine the optimal math by selecting the candidate pattern with maximum similarity score to $O^{'}$ as the final recognition result.

Numerical examples from literature are used to validate similarity measure proposed in this paper. Suppose that there are three mineral fields, donated as A, B, C. These fields can each be described by five characteristics s_1, s_2, s_3, s_4, s_5, with the weight vector of $(0.25, 0.2, 0.15, 0.18, 0.22)^T$. The evaluation values of the three mineral fields under five standards are shown in Table 2.

Table 2 Decision values for mineral field recognition

f	A	B	C	O
s_1	([0.37, 0.49], [0.03, 0.11], [0.34, 0.40])	([0.23, 0.33], [0.13, 0.20], [0.11, 0.19])	([0.12, 0.35], [0.07, 0.18], [0.22, 0.32])	([0.20, 0.28], [0.07, 0.15], [0.31, 0.50])
s_2	([0.07, 0.23], [0.11, 0.29], [0.21, 0.33])	([0.13, 0.31], [0.02, 0.13], [0.22, 0.44])	([0.26, 0.44], [0.02, 0.08], [0.16, 0.27])	([0.33, 0.51], [0.02, 0.17], [0.20, 0.36])
s_3	([0.27, 0.36], [0.09, 0.19], [0.13, 0.18])	([0.09, 0.19], [0.17, 0.31], [0.22, 0.36])	([0.14, 0.19], [0.21, 0.32], [0.36, 0.41])	([0.17, 0.37], [0.04, 0.14], [0.22, 0.36])
s_4	([0.09, 0.43], [0.12, 0.21], [0.14, 0.35])	([0.12, 0.21], [0.08, 0.13], [0.24, 0.49])	([0.13, 0.19], [0.08, 0.22], [0.48, 0.58])	([0.12, 0.24], [0.11, 0.21], [0.36, 0.49])
s_5	([0.16, 0.48], [0.14, 0.30], [0.01, 0.11])	([0.13, 0.34], [0.01, 0.23], [0.31, 0.42])	([0.28, 0.38], [0.10, 0.20], [0.14, 0.40])	([0.15, 0.26], [0.09, 0.17], [0.43, 0.56])

Assume there exists an ideal mineral field O, and therefore, it is needed to determine which mineral field is closest to O. Experts will evaluate each mineral field based on the five standards. The similarity values of each mineral field with O

under the weight vector $\omega=(0.25, 0.2, 0.15, 0.18, 0.22)^T$ were calculated using the similarity measure formula proposed in this paper, as shown in Table 3.

Table 3 Similarity Measure of OA, OB, OC

SM	(O, A)	(O, B)	(O, C)	result
S	0.8704	0.9063	0.9177	C>B>A
S_{Li}	0.8099	0.9001	0.9269	C>B>A
S_{CSM}^1	0.2525	0.2847	0.3002	C>B>A
S_{CSM}^2	0.3372	0.3681	0.3726	C>B>A
S_{CsSM}^1	0.8748	0.9604	0.9443	B>C>A
S_{CsSM}^2	0.7700	0.9140	0.9045	B>C>A
S_{CsSM}^3	0.8728	0.9263	0.9449	C>B>A
S_{CsSM}^4	0.6115	0.8158	0.8205	C>B>A
S_{CtSM}^1	0.6607	0.7634	0.7337	C>B>A
S_{CtSM}^2	0.6033	0.6874	0.7109	C>B>A
S_{StSM}	0.6235	0.8214	0.7841	B>C>A
S_{GSM}	0.7426	0.7702	0.8132	C>B>A
S_{DSM}^1	0.7670	0.9010	0.9004	B>C>A
S_{DSM}^2	0.8108	0.8973	0.9118	C>B>A
S_{DSM}^3	0.8010	0.8902	0.9260	C>B>A
S_{DSM}^4	0.3038	0.2822	0.3195	C>B>A

Note: S is the entropy-based similarity measure proposed in this paper, others are existing similarity measures.

From the results in Table 3, it can be seen that 70% results show that C is the most similar to the ideal mineral filed O, and there are also four instances where B is shown to be the most similar to O. Besides, the ranking outcomes computed by S_{CSM}^1, S_{CSM}^2 and S_{DSM}^4 are also unpersuasive because the similarity values are too low to prove that two IvPFSs are similar. It shows that the similarity measure S proposed in this paper is effective and consistent with reality.

4 CONCLUSIONS AND OUTLOOKS

This paper focuses on constructing entropy and similarity measures for IvPFS. It represents an IvPFS as a tetrahedron in a three-dimensional coordinate system and selects 11 representative boundary points based on interval upper and lower bounds. The proposed method develops a novel entropy measure by computing the arithmetic mean of distances between these points across two tetrahedrons. It also defines the fuzziest IvPFS $\mathscr{M}$ and introduces a corresponding similarity measure based on the entropy between the aggregated IvPFS $N(A, B)$ (generated by aggregation operators) and $\mathscr{M}$. Compared with existing measures, the proposed entropy and similarity measures exhibit three key advantages: simplified computational procedures, reduced parameter dependency, and strict adherence to similarity axioms. Finally, they are applied to the field of pattern recognition, providing robust metric tools for both theoretical exploration and real-world implementations of IvPFS.

However, the idea based on the arithmetic mean and Euclidean distance is the most fundamental method for constructing similarity measure. In the future, more reasonable construction methods will continue to emerge to solve complex practical problems.

COMPETING INTERESTS

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