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MATHEMATICAL MODELING AND HIGH-ORDER FINITE DIFFERENCE METHODS FOR SPATIAL FRACTIONAL FITZHUGH-NAGUMO EQUATIONS

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Abstract: Mathematical modeling of the spatial fractional-order FitzHugh-Nagumo equations provides a critical framework for describing the propagation of electrical potentials in heterogeneous cardiac tissues, a problem that continues to attract significant research attention. Due to the geometrically irregular cross-sections of cardiac tissue, numerical solutions on conventional regular or approximately irregular domains remain limited, and analytical solutions are generally unavailable. In this study, we employ a high-order finite difference method in space coupled with a fourth-order Runge-Kutta scheme in time to solve this nonlinear fractional-order system. Numerical experiments are conducted to validate the high-order convergence and computational stability of the proposed numerical approach.
Keywords: Riesz fractional derivative; FitzHugh-Nagumo model; Finite difference method; The fourth-order explicit Runge-Kutta method

1 INTRODUCTION

Differential equations can describe phenomena ranging from simple linear growth or decay to complex dynamical systems, such as fluid flow, heat conduction, population dynamics, and electrical circuits. As fundamental tools for studying real-world processes, differential equations can be analyzed through analytical methods or numerical computations to reveal the evolution of systems over time[1]. Hodgkin and Huxley proposed a mathematical model describing the transmission of neuronal action potentials, which revealed the mechanism of neural signal conduction. Due to the complexity of the differential equations involved, FitzHugh and Nagumo subsequently proposed a simpler integer-order FitzHugh-Nagumo (FHN) model. Known for its concise mathematical form and accurate depiction of the core dynamics of excitable systems, it has become a fundamental model in neuroscience, cardiac electrophysiology, nonlinear dynamics, and engineering[2].

However, classical integer-order models have limitations in describing the inherent temporal memory dependence and spatial correlation complexity of biological systems. The introduction of fractional-order FHN models effectively addresses this issue, significantly enhancing the biophysical realism and adaptability of the models[3,4]. In recent years, research on fractional-order FHN models has progressed: In 2019, Prakash and Kaur used the homotopy perturbation transform technique to study fractional-order FHN equations for neural impulse transmission, validating the algorithm's high-order convergence and errors[5]; In 2021, Wang et al. used the finite difference method to study the fractional-order FHN monodomain model on moving irregular domains, addressing challenges in cardiac electrophysiological simulation[6]; In 2025, Kumar and Erturk used generalized Caputo fractional derivatives to construct a fractional-order modified FHN neuron model, employed the L1 predictor-corrector method for numerical solutions and conducted error and stability analysis, studying periodic spiking, chaos, and other neuronal electrical activities generated by the model[7].

This paper studies the following one-dimensional spatial fractional FHN equation with Riesz fractional derivative[4]

$$\frac{\partial u}{\partial \mathrm{t}} = -K_u(-\Delta)^{\frac{\alpha}{2}}u - u(1-u)(u-\mu) - v\,, \tag{1}$$

$$\frac{\partial v}{\partial t} = \varepsilon(\beta u - v), \tag{2}$$

$$u(x,0) = u_0(x), v(x,0) = v_0(x). \tag{3}$$

where $(x,t) \in \Omega \times [0,T]$, $\Omega = (a,b)$, $1 < \alpha \le 2$, ε, μ and β are constants, K_u is the diffusion coefficient. The one-dimensional Riesz fractional derivative is defined as[8]

$$-(-\Delta)^{\frac{\alpha}{2}}u = \frac{\partial^\alpha u}{\partial |x|^\alpha} = -\frac{1}{2cos(\pi\alpha/2)}\left({}_xD_L^\alpha u + {}_xD_R^\alpha u\right),$$

where ${}_xD_L^\alpha u$ and ${}_xD_R^\alpha u$ are Riemann-Liouvillefractional operators, defined respectively as[8]:

$$ {}_xD_L^{\alpha}u(x,t)=\frac{1}{\Gamma(2-\alpha)}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\int_a^x(x-\tau)^{1-\alpha}u(\tau,t)\mathrm{d}\tau\right), $$

$$ {}_xD_R^{\alpha}u(x,t)=\frac{1}{\Gamma(2-\alpha)}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\int_x^b(\tau-x)^{1-\alpha}u(\tau,t)\mathrm{d}\tau\right). $$

The finite difference method, due to its high approximation accuracy for fractional derivatives and ease of implementing discrete schemes, has become an important numerical tool for solving spatial fractional differential equations. For the discretization of the Riesz fractional derivative, Huang et al. proposed a fourth-order accuracy finite difference scheme, reducing the truncation error of fractional operator discretization and providing a high-order numerical method for solving high-dimensional fractional reaction-diffusion equations[9]; Han et al. used this method to handle the Riesz fractional diffusion term, successfully solved the fractional Gray-Scott model, and validated its applicability in nonlinear systems[10]. The fourth-order explicit Runge-Kutta method, due to its high temporal accuracy and good stability, is widely used for solving semi-discrete ordinary differential equation systems. In the study of integer-order FHN equations, Chin used this method for temporal discretization, confirming that the fourth-order explicit Runge-Kutta method can accurately capture the system's dynamic behavior and has good stability[11]. This paper will apply the finite difference method combined with the fourth-order explicit Runge-Kutta method to discretize the one-dimensional spatial fractional FHN equation with Riesz fractional derivative and prove the stability, convergence, and error analysis of the numerical method.
The content arrangement of this paper is as follows: Section 1 discretizes the one-dimensional Riesz fractional derivative in the spatial fractional FHN equation using the finite difference method, transforming the original partial differential equation system into a semi-discrete ordinary differential equation system, then performs temporal discretization on the ODE system to obtain a fully discrete numerical scheme, and proves the stability and convergence of the numerical method. Section 2 validates the theoretical analysis results through several numerical experiments; Section 3 provides a brief conclusion.

2 NUMERICAL SCHEME AND ITS PROPERTIES

2.1 Numerical Scheme

2.1.1 Spatial discretization

This subsection uses finite difference approximation for the fractional Laplacian to obtain a semi-discrete ordinary differential system. Let the spatial step size $h=(b-a)/M$, where is a constant. The spatial grid points are defined as

$$ \Omega_h=\{x_i \mid x_i=a+ih, i=0,1,\dots,M-1\}, $$

denote $L^{4+\alpha}(\mathbb{R}):=\{f\in L^{4+\alpha}(\mathbb{R})\mid \int_{-\infty}^{\infty}(1+|\xi|)^{4+\alpha}|\hat{f}(\xi)|\mathrm{d}\xi<\infty\}$, $\hat{f}(\xi)=\int_{-\infty}^{\infty}f(x)e^{i\xi x}\mathrm{d}x$, then the numerical scheme for approximating the Riesz fractional derivative $-(-\Delta)^{\alpha/2}u$ is given by the following lemma.

Lemma 2.1[4,9] Assuming $f(x)\in L^{4+\alpha}(\mathbb{R})$, then

$$ \frac{\partial^{\alpha} f}{\partial|x|^{\alpha}}=-\delta_h^{\alpha}f(x)+O(h^4), $$

where $\delta_h^{\alpha}f(x)=\frac{1}{h^{\alpha}}\sum_{k=-\infty}^{\infty}\omega_k^{\alpha}f(x-kh)$, and the weight coefficients ω_k^{α} are defined as

$$ \omega_k^{\alpha}=\begin{cases}\frac{4}{3}g_k^{\alpha}-\frac{1}{3\cdot 2^{\alpha}}g_{k/2}^{\alpha}, & k \ \ is \ \ even\\ \frac{4}{3}g_k^{\alpha}, & k \ \ is \ \ odd\end{cases}, $$

where $g_k^{\alpha}=\frac{(-1)^k\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}$.

Let $u_i=u(x_i,t)$, $v_i=v(x_i,t)$, performing finite difference approximation on equations (1)(2), we obtain

$$ \frac{\partial u_i}{\partial \mathrm{t}}=-K_u\delta_h^{\alpha}u_i-u_i(1-u_i)(u_i-\mu)-v_i \ , \tag{4} $$

$$ \frac{\partial v_i}{\partial \mathrm{t}}=\varepsilon(\beta u_i-v_i), \tag{5} $$

where

$$\delta_h^\alpha u_i = \frac{1}{h^\alpha}\sum_{k=0}^{i} g_k^\alpha u_{i-k}\,,\ g_k^\alpha = \left(-1\right)^k C\left(\alpha,k\right),\ C\left(\alpha,k\right) = \alpha\left(\alpha-1\right)\left(\alpha-2\right)\cdots\left(\alpha-k+1\right)/k!.$$

Let $U=(u_0,u_1,\ldots,u_{N-1})^T$, $V=(v_0,v_1,\ldots,v_{N-1})^T$, equations (4)(5) transform into the following semi-discrete ordinary differential equation system

$$\begin{cases}\dfrac{dU}{dt} = -K_u\delta_h^\alpha U - U(1-U)(U-\mu) - V,\\ \dfrac{dV}{dt} = \varepsilon(\beta U - V),\end{cases} \tag{6}$$

further let $Q=[U,V]^T$, $F(Q)=[-K_u\delta_h^\alpha U - U(1-U)(U-\mu)-V, \varepsilon(\beta U - V)]^T$, then we have

$$\frac{dQ}{dt} = F(Q). \tag{7}$$

2.1.2 Temporal Discretization

Next, the fourth-order Runge-Kutta method is used to solve the ordinary differential equation (6). Let the time step size $\tau = T/N$, where N is a constant. Spatial and temporal grid points are formed by $\Omega_h^\tau = \Omega_h \times \Omega_\tau$, where $\Omega_\tau = \left\{t_n \mid t_n = n\tau, n=0,1,...,N\right\}$, applying the fourth-order explicit Runge-Kutta method to equation (7) yields:

$$\begin{cases} k_1 = F(Q^n),\\ k_2 = F(Q^n + 2\tau k_1),\\ k_3 = F(Q^n + 2\tau k_2),\\ k_4 = F(Q^n + \tau k_3),\\ Q^{n+1} = Q^n + \tau\Phi\left(Q^n,\tau\right),\end{cases} \tag{8}$$

where $Q^0 = [U_0, V_0]^T$, $\Phi\left(Q^n,\tau\right) = 1/6(k_1 + 2k_2 + 2k_3 + k_4)$.

2.2 Properties of the Numerical Scheme

2.2.1 Stability

Define $\|\bullet\|_2$ as the l_2 norm, the l_2 norm of Q^n is defined as:

$$\left\|Q^n\right\|_2 = \sqrt{\left\|U^n\right\|_2^2 + \left\|V^n\right\|_2^2} = \sqrt{\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}\left((u_i^n)^2 + (v_i^n)^2\right)}.$$

Theorem 2.2[10,12] If $\Phi\left(Q,\tau\right)$ satisfies the Lipschitz condition in Q , then this numerical method is stable, i.e., there exists a positive constant τ 、 h independent of C , such that for any time step $n(0 \le n\tau \le T)$, $\left\|Q^n\right\|_2 \le C\left\|Q^0\right\|_2$.

Proof By the premise of the theorem, $\Phi\left(Q,\tau\right)$ satisfies the Lipschitz condition in Q , i.e., there exists a Lipschitz constant $L_F > 0$, for any two vectors Q_1、 Q_2 , we have

$$\left\|\Phi\left(Q_1,\tau\right) - \Phi\left(Q_2,\tau\right)\right\|_2 \le L_F \| Q_1 - Q_2 \|_2.$$

Let $Q_1 = Q^n$, $Q_2 = 0$ (zero vector), then it can be deduced that

$$\left\|\Phi\left(Q^n,\tau\right)\right\|_2 \le L_F \| Q^n \|_2 + \left\|\Phi\left(0,\tau\right)\right\|_2,$$

however, since at the initial time Q^0 is obtained by discretizing the given initial conditions $u_0\left(x\right)$ 、 $v_0\left(x\right)$, its norm $\left\|Q^0\right\|$ is bounded. By Lemma 2.1 and its derivation, δ_h^α is a fourth-order accuracy approximation, and the norm of its corresponding matrix l_2 is bounded, i.e., $\left\|\delta_h^\alpha U\right\|_2 \le C_\alpha h^{-\alpha}\left\|U\right\|_2$, C_α is a constant only related to α . It is concluded that the linear term $F\left(Q\right)$ in (containing $\delta_h^\alpha U$) is bounded, and the nonlinear term $-U(1-U)(U-\mu)$ is locally bounded within a bounded domain, so $\left\|\Phi\left(0,\tau\right)\right\|_2$ can be controlled by a constant independent of τ、 h, temporarily denoted here as C_0, i.e., $\left\|\Phi\left(Q^n,\tau\right)\right\|_2 \le L_F\left\|Q^n\right\|_2 + C_0$.

Taking the l_2 norm of the fifth equation in the fully discrete scheme (9), by the triangle inequality

$$\left\|Q^{n+1}\right\|_2 = \left\|Q^n + \tau\Phi\left(Q^n,\tau\right)\right\|_2 \le \left\|Q^n\right\|_2 + \tau\left\|\Phi\left(Q^n,\tau\right)\right\|_2,$$

Substituting the above estimate for $\left\|\Phi\left(Q^n,\tau\right)\right\|_2$, we obtain

$$\left\|Q^{n+1}\right\|_2 \le \left\|Q^n\right\|_2 + \tau(L_F\left\|Q^n\right\|_2 + C_0) = (1+\tau L_F)\left\|Q^n\right\|_2 + \tau C_0,$$

Next, analyze the evolution trend of the norm through iteration: at initial time $\mathrm{n}=0$, $\left\|Q^0\right\|_2$ is bounded (denoted as $C_1 = \left\|Q^0\right\|_2$);

When $n=1$, $\left\|Q^1\right\|_2 \le (1+\tau L_F)C_1 + \tau C_0$;

When $n=2$, $\left\|Q^2\right\|_2 \le (1+\tau L_F)\left\|Q^1\right\|_2 + \tau C_0 \le (1+\tau L_F)^2 C_1 + \tau C_0(1+(1+\tau L_F))$;

And so on, for any n, we have: $\left\|Q^n\right\|_2 \le (1+\tau L_F)^n C_1 + \tau C_0 \sum_{k=0}^{n-1}(1+\tau L_F)^k$,

using the geometric series sum formula $\sum_{k=0}^{n-1}(1+\tau L_F)^k = \dfrac{(1+\tau L_F)^n - 1}{\tau L_F}$, ($L_F \ne 0$, if $L_F = 0$ then $F\left(Q\right)$ is a constant vector, stability obviously holds), substituting yields:

$$\left\|Q^n\right\|_2 \le (1+\tau L_F)^n C_1 + \frac{C_0}{L_F}[(1+\tau L_F)^n - 1].$$

By the properties of the exponential function, for any $x>0$, positive integer m, have $(1+x)^m \le e^{mx}$, here $x = \tau L_F$ 、 $m=n$, and $n\tau \le T$ (final time fixed), so $(1+\tau L_F)^n \le e^{n\tau L_F} \le e^{TL_F}$ (let $C_2 = e^{TL_F}$, independent of τ、 h).

Since $C_2 = e^{TL_F}$ is always constant, and $\dfrac{C_0}{L_F}[(1+\tau L_F)^n - 1] \le \dfrac{C_0}{L_F}(C_2 - 1)$ (denoted as C_3, independent of τ、 h).

Therefore: $\left\|Q^n\right\|_2 \le C_2 C_1 + C_3$.

Let $C = \max\{C_2C_1 + C_3, C_1\}$ (independent of τ、 h), then for any n exist $\left\|Q^n\right\|_2 \le C$, and since $C_1 = \left\|Q^0\right\|_2$, C can be further adjusted such that $\left\|Q^n\right\|_2 \le C\left\|Q^0\right\|_2$ (if $\left\|Q^0\right\|_2 = 0$ then the numerical solution is always zero, if $\left\|Q^0\right\|_2 > 0$, then C can be taken as $\max\left\{C_2 + C_3/C_1, 1\right\}$), i.e., the l_2 norm of the numerical solution vector remains bounded during the evolution process, hence this numerical method is stable.

2.2.2 Convergence and error analysis

Theorem 2.3 Let the global error be $e_n = Q(t_n) - Q^n$, combining Lemma 2.1 and equation (8) obtained by the fourth-order Runge-Kutta method, if is satisfied $\tau/h^\alpha \le 1$, then there exist positive constants C_4, C_5, (independent of τ and h), such that $\left\|e_n\right\| \le C_4 h^4 + C_5 \tau^4$.

Proof Let the projection vector of the exact solution $u\left(x,t\right)$、 $v\left(x,t\right)$ at grid point Ω_h be $Q(t) = [U(t)^T, V(t)^T]^T$, where $U(t), V(t)$ are composed of $u\left(x_i,t\right)$ and $\mathrm{v}\left(x_i,t\right)$ respectively.

By Lemma 2.1, the spatial discretization truncation error is $O(h^4)$, i.e.

$$\left\|-\delta_h^\alpha u(x) - \frac{\partial^\alpha u}{\partial\left|x\right|^\alpha}\right\| \le K_1 h^4,$$

where K_1 is a constant, $u \in L^{4+\alpha}(\mathbb{R})$. The exact solution $Q_h\left(t\right)$ of the semi-discrete system (8) satisfies

$$\| Q(t)-Q_h(t) \| \le C_s h^4, \forall t \in [0,T],$$

where $C_s > 0$ is a constant independent of h, t.

The local truncation error of the fourth-order Runge-Kutta method is $O(\tau^5)$, i.e.

$$\| Q_h(t_{n+1})-Q_h(t_n)-\tau\Phi(Q_h(t_n),\tau) \| \le K_2\tau^5,$$

where $K_2 > 0$ is a constant independent of τ. Under the stability condition $\tau/h^\alpha \le 1$, by the Lipschitz condition we obtain $\| Q_h(t_n)-Q^n \| \le C_t\tau^4, \forall t_n \le T$,where $C_t > 0$ is a constant independent of τ, n.

The global error is decomposed as: $e_n = Q(t_n)-Q^n = \left[Q(t_n)-Q_h(t_n)\right]+\left[Q_h(t_n)-Q^n\right]$,

applying the triangle inequality: $\| e_n \| \le \| Q(t_n)-Q_h(t_n) \| + \| Q_h(t_n)-Q^n \| \le C_s h^4 + C_t\tau^4$, taking $C_4 = C_s$, $C_5 = C_t$, yields the conclusion.

3 NUMERICAL EXPERIMENTS

3.1 Experiment 1

Consider the following one-dimensional FHN equation[13,14]

$$\frac{\partial u}{\partial t} = u_{xx} + u(1-u)(u-\mu),$$

whose exact solution is $u(x,t) = \frac{1}{2}+\frac{1}{2}\tanh[k(x-ct)]$, where $k = \frac{1}{2\sqrt{2}}$, $c = \frac{2^\mu - 1}{\sqrt{2}}$. The initial condition is $u(x,0) = \frac{1}{2}+\frac{1}{2}\tanh\left(\frac{x}{2\sqrt{2}}\right)$.

Define the errors E_2 and E_∞ as

$$E_2 = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left[u(x_j,t)-u^*(x_j,t)\right]^2},\ E_\infty = \max_{1\le j\le N}\left|u(x_j,t)-u^*(x_j,t)\right|,$$

where $u(x_j,t)$ and $u^*(x_j,t)$ are the numerical and exact solutions respectively.

In Table 1, taking $\mu = 0.75$, $h = 0.1$, for different values of τ, calculate the E_2 and E_∞ errors from time 0.01 to 50 respectively. From Table 1, it can be seen that the errors decrease over time and with decreasing time step size. In Table 2, for different values of h, taking $\mu = 0.75$, $\tau = 10^{-5}$, calculate the E_2 and E_∞ error norms and from time 0.01 to 50 respectively. From Table 2, it can be seen that the error norms decrease over time and with decreasing spatial step size. The solution obtained by this method is shown in Figure 1, where $h = 0.1$, $\tau = 5\times10^{-3}$, μ are taken as 0.25, 0.50, 0.75, 0.10 respectively.

Table 1 Error Norms under Different for τ, $h = 0.1$, $\mu = 0.75$

t	$\tau = 10^{-3}$		$\tau = 10^{-4}$		$\tau = 10^{-5}$	
	E_2	E_∞	E_2	E_∞	E_2	E_∞
0.01	2.55e-05	5.53e-05	2.56e-06	5.53e-06	3.31e-07	5.53e-07
1	4.21e-05	5.93e-05	3.52e-06	5.93e-06	3.72e-06	4.26e-06
10	1.67e-05	3.06e-05	1.70e-06	3.06e-06	4.24e-07	4.85e-07
50	1.23e-09	1.89e-09	4.23e-10	3.93e-10	3.58e-10	3.45e-10

Table 2 Error Norms under Different for h, $\tau = 10^{-5}$, $\mu = 0.75$

t	$h = 0.1$		$h = 0.05$		$h = 0.01$	
	E_2	E_∞	E_2	E_∞	E_2	E_∞
0.01	3.31e-07	5.53e-07	2.00e-07	5.53e-07	1.25e-07	5.53e-07
1	3.72e-06	4.26e-06	7.65e-07	8.73e-07	3.60e-07	5.93e-07

10	4.24e-07	4.85e-07	1.79e-07	3.06e-07	1.40e-07	3.06e-07
50	3.58e-10	3.45e-10	9.46e-11	9.03e-11	1.10e-11	1.89e-11

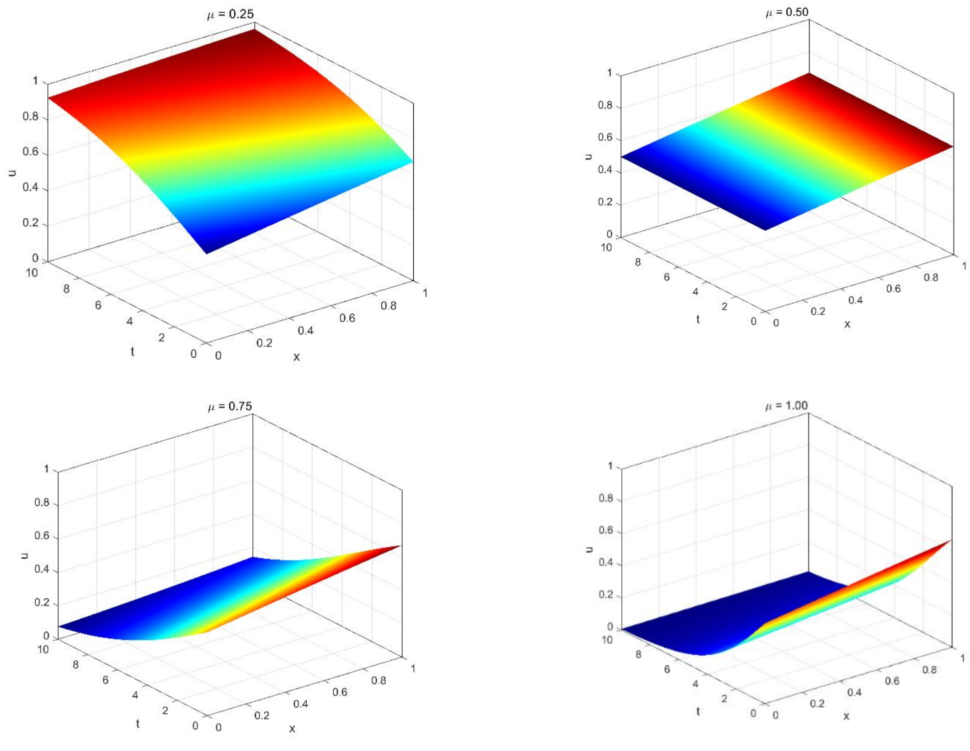

Figure 1 Numerical Solution under Different for μ, $h = 0.1$, $\tau = 5\times10^{-3}$

4 CONCLUSION

This paper proposes a numerical solution scheme combining high-order finite difference method and fourth-order explicit Runge-Kutta method for the one-dimensional spatial fractional FitzHugh-Nagumo model with Riesz fractional derivative. The paper provides rigorous theoretical analysis of the stability and convergence of the numerical method, proving that under certain conditions the scheme is stable and has fourth-order accuracy in both time and space.

Numerical experimental results show that this method exhibits good numerical performance under different fractional orders, time step sizes, and spatial step sizes. Errors decrease significantly with decreasing step sizes, validating the correctness of the theoretical analysis. The intuitive display of the evolution behavior of numerical solutions under different parameters further confirms the effectiveness and reliability of this method in simulating the dynamic processes of fractional FHN systems. This work also serves as an illustrative case in the curriculum reform project : Restructuring and Practical Research on an Interdisciplinary PBL Course System for Mathematical Modeling Driven by Generative AI, demonstrating how advanced computational methods can be integrated into modeling education.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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REFERENCES

[1] Han C, Wang Y L, Li Z Y. Novel patterns in a class of fractional reaction–diffusion models with the Riesz fractional derivative. Mathematics and Computers in Simulation, 2022, 202, 149–163.

[2] Zhang L, Sun H W. Numerical solution for multi-dimensional Riesz fractional nonlinear reaction–diffusion equation by exponential Runge–Kutta method. Journal of Applied Mathematics and Computing, 2019, 60 (1-2): 357-378.

[3] Zeng F, Liu F, Li C P, et al. A Crank--Nicolson ADI spectral method for a two-dimensional Riesz space fractional nonlinear reaction-diffusion equation. SIAM Journal on Numerical Analysis, 2014, 52(6): 2599-2622.

[4] Li X, Han C, Wang Y. Novel Patterns in Fractional-in-Space Nonlinear Coupled FitzHugh-Nagumo Models with Riesz Fractional Derivative. Fractal and Fractional, 2022, 6(3), 136.
[5] Prakash A, Kaur H. A reliable numerical algorithm for a fractional model of Fitzhugh-Nagumo equation arising in the transmission of nerve impulses. Nonlinear Engineering, 2019, 8(1):719-727.
[6] Wang Y H, Feng X L, Li H. Ghost structure finite difference method for fractional FitzHugh-Nagumo monodomain model on moving irregular domains. Journal of Computational Physics, 2021, 424: 110862.
[7] Kumar P, Erturk V S. A fractional-order improved FitzHugh-Nagumo neuron model. Chinese Physics B, 2025, 34(1): 018704.
[8] Lee H G. A second-order operator splitting Fourier spectral method for fractional-in-space reaction–diffusion equations. Journal of Computational and Applied Mathematics, 2018, 333: 395-403.
[9] Huang Y Y, Qu W, Lei S L. On τ-preconditioner for a novel fourth-order difference scheme of two-dimensional Riesz space-fractional diffusion equations. Computers & Mathematics with Applications, 2023, 145: 124-140.
[10] Han C, Wang Y L, Li Z Y. A High-Precision Numerical Approach to Solving Space Fractional Gray-Scott Model. Applied Mathematics Letters, 2022, 125: 107759.
[11] Chin P W M. The effect on the solution of the FitzHugh-Nagumo equation by the external parameter using the Galerkin method. Journal of Applied Analysis & Computation, 2023, 13(4): 1983-2005.
[12] Han C, Wang Y L, Li Z Y. Numerical solutions of space fractional variable-coefficient KdV–modified KdV equation by Fourier spectral method. Fractals, 2021, 29(08): 2150246.
[13] Zhang J J, Chen Y M, Wang X M. High-precision compact difference method for the FitzHugh-Nagumo equation. Journal of China West Normal University (Natural Science Edition), 2021, 42(2): 126–132.
[14] Luo Y, Song L Y. Solve the generalized Fitzhugh-Nagumo equation with time-varying coefficients based on the Chebyshev spectral method. Journal of Jiamusi University (Natural Science Edition), 2025, 43(3): 166–170.