Eurasia Journal of Science and Technology
Print ISSN: 2663-1024
Online ISSN: 2663-1016
DOI: https://doi.org/10.61784/ejst3132

CAN CONSUMER'S IDENTITY MANAGEMENT AFFECT PLATFORM'S BEHAVIOR-BASED PERSONALIZED PRICING?

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Abstract: With the development of big data and platform economy, platforms can easily gather consumer information through first-period purchase and exercise personalized pricing. When consumers realize it, they may engage in identity management to bypass the platform's attempt to price discriminate. Therefore, it is significant to clarify the impact of identity management on platform's behavior-based price discrimination, and whether platforms continue to compete consumers if they realize that consumers may engage in identity management. We introduce the identity management into the standard framework of behavior-based price discrimination, then we analyze and compare equilibrium results with and without identity management. The main findings are as the follows: (i) when consumers are active in identity management, the competition between platforms is intensified in the information collection stage, but it is softened in the information application stage, and platforms can exercise perfect price discrimination to consumers. (ii) The consumer identity management leads to lower consumer surplus and higher social welfare. When consumers become farsighted, platforms can be better off. (iii) platforms still have incentives to compete with consumers and gather consumer information, even though platforms predict that consumers are active in the identity management. (iv) Compared to one-sided market, the consumer identity management in two-sided market makes platforms more willing to implement perfect price discrimination.
Keywords: Behavior-based price discrimination; Personalized pricing; Identity management; Platform

1 INTRODUCTION

With the development of big data, algorithms, and the digital platform economy, platform companies can increasingly easily obtain information about consumers. For example, ride-hailing platform companies can use price competition to offer consumers a ride for the first time. After that, they can not only determine whether the consumer belongs to their target demographic but also track the consumer's ride history using tools like cookies and web beacons, thereby obtaining a wealth of preference information. After platform companies obtain information about consumers, they can tailor prices for each consumer based on their services, i.e., the personalized pricing. The Personalized pricing is a form of price discrimination, where platform companies set the same service with different prices for each consumer, which is rare in traditional markets. The improvement in the quality and quantity of information has enabled platform companies to offer more personalized prices, which leads to emergence of real-time personalized pricing[1-2]. The behavior of platforms, which involves collecting personal-level information about consumers by tracking their ride records and using it for personalized pricing, is known as behavior-based price discrimination (BBPD)[3]. Behavioral price discrimination by platform companies is already widespread[4]. For example, Uber's "route-based pricing" charges based on the consumer's expected willingness to pay. The frequent occurrence of big data price discrimination by platform companies in practice is a real-world manifestation of behavioral price discrimination. Didi and Shouqi ride-hailing previously implemented big data price discrimination[5]. In digital platform companies, behavioral price discrimination will be more easily implemented, and the prices set for each consumer will be more refined and personalized.

Consumers are aware of that their personal information is collected and they may experience price discrimination with the increasingly common phenomenon of information collecting. To protect their privacy or seek services of lower prices, some consumers actively take measures to thwart platform companies' attempts at price discrimination. For example, consumers delete or block cookies that track them, create new accounts for new transactions, and maintain multiple virtual identities, etc. Acquisti refers to this proactive behavior of consumers in protecting their personal information as the identity management[6], and consumers who implement the identity management are called active consumers. However, consumers may need to invest time, effort, or money in the identity management. Therefore, some consumers face significant costs when managing their identities, making it difficult for them to take measures to counteract the price discrimination by platform companies, and they are referred to as passive consumers. In addition, platform companies may also set up barriers that prevent consumers from choosing between the personalized prices platforms set and the uniform prices set for new consumers, thereby making it impossible for consumers to manage their identities. For example, when active consumers repeatedly use the Didi platform, they can see the personalized prices offered to them on the platform. Thus, they can then re-register a new account to observe the uniform prices offered to new consumers by the platform, and they can also understand the uniform prices offered by the Shouqi platform before making their decision. However, passive consumers may be deterred by the time cost, and they are unwilling to spend time re-registering an account to observe the uniform prices on the Didi platform.

Consumer identity management can significantly affect the pricing strategies of platforms. Therefore, the literature categorizes identity management into three types based on the sequence in which consumers take protective measures: Ex-post management, concurrent management, and ex-post management. Pre-emptive identity management refers to consumers taking measures to prevent platforms from collecting information when they first make a purchase, such as refusing to accept cookies. Concurrent identity management refers to actions after their initial purchase but before the platform sets a price for repeat purchases, in order to avoid identification by the platform. Ex-post identity management refers to actions after observing the price set by the platform for repeat purchases. Most previous literature has mainly considered Pre-emptive identity management[7-8], Concurrent identity management[9], and the scenario of monopolistic firms. However, consumers typically choose whether to engage in identity management only after observing the price set by the platform for repeat purchases in practice, as they will take such measures only if it is in their interest. Therefore, ex-post identity management is more in line with real-world situations. Research on ex-post identity management is generally concentrated in single-sided markets, but personalized pricing and identity management are more prevalent in two-sided markets. Moreover, if platforms anticipate that consumers engage in identity management, they may alter their strategies based on behavioral price discrimination. Thus, it is crucial to investigate the impact of ex-post identity management on platforms' behavioral price discrimination from a dynamic perspective. Note that the dynamic perspective in this paper refers to a model that includes the phase where platforms obtain collect information from transactions with consumers.

This paper mainly addresses the following questions: (1) Whether the pricing strategies of platforms based on behavioral price discrimination, platform profits, consumer surplus, and social welfare are influenced by consumer identity management? (2) If platform companies anticipate that consumers engage in identity management, whether platforms still have incentives to collect consumer information for BBPD? That is to say, whether platforms are prevented from BBPD when all consumers consider identity management? (3) How do network externalities affect consumer identity management and BBPD of platforms?

To address the aforementioned questions, this paper constructs a two-period dynamic model with horizontal differentiation for a dual oligopoly. In the first period, platform companies engage in standard Hotelling price competition; in the second period, they compete with personalized pricing for repeat consumers and uniform pricing for new consumers. Besides using the service in each period, consumers manage their identities in the second period after observing the prices of platforms for repeat consumers. Service providers offer services to consumers in both periods. First, we present the competitive platforms' equilibrium based on behavioral price discrimination when consumer post-identity management does not exist from a dynamic perspective. We then investigate the equilibrium of platforms' behavioral price discrimination when the consumer identity management exists. Furthermore, we compare the equilibrium with and without consumer identity management. Finally, we extend the model by considering costs of consumer identity management to verify the robustness of the results. The results indicates that (1) The BBPD of platforms are affected by the consumer identity management. The consumer identity management will intensify the competition between platforms during the phase where platforms collect consumer information, which leads to lower prices. When platforms show due attention to the future, and utilities of services for consumers are limited. Consumer identity management weakens competition during the phase where platforms collect consumer information, enabling them to implement BBPD, extracting all consumer surplus and raising prices.; (2) Consumer surplus decreases in the consumer identity management, but social welfare increases in the consumer identity management. When consumers value future consumption, identity management benefits all platforms; (3) Even if platforms anticipate consumer identity management, they still have incentives to compete for consumers and collect consumer information to implement BBPD; (4) Unlike the scenario in one-sided markets, network externalities strengthen the willingness of platforms to implement the perfect BBPD in two-sided markets. Later on, the extension of this paper shows that the equilibrium does not depend on the cost of identity management if the cost of consumer identity management is less than a certain threshold.

The contributions of this paper are as follows: (1) Unlike previous literature on consumer identity management that focuses on one-sided markets, this paper introduces consumer identity management into the framework of BBPD in two-sided platforms, thereby expanding the research on consumer identity management and BBPD. (2) We clarify whether the pricing strategies of platforms are influenced by consumer identity management, and the impact of ex-post consumer identity management on platforms' pricing and the surplus. This paper provides a theoretical reference for consumer privacy policies and the management of consumer information by platforms. (3) It is clear that even if platforms anticipate that consumers will conduct identity management, they still have incentives to compete for consumers, collect consumer information, and implement BBPD. (4) compared with one-sided markets, consumer identity management is more likely to enable platforms to implement perfect price discrimination in the two-sided market.

The remainder of the paper is as follows: Section 2 is Literature Review. Section 3 is the model setup and the benchmark results of BBPD by platforms without consumer identity management. And we analyze the pricing of platforms with consumer identity management. In Section 5, we compare the equilibrium with and without consumer identity management. Section 6 is the extension. Finally, the last section is the discussion and conclusion.

2 LITERATURE REVIEW

A lot of literature concerning platforms' collection of consumer information for price discrimination has developed. The previous conclusions are that when platforms gather more consumer information, it facilitates price discrimination and intensify the competition among platforms, which may undermine the profits of platforms. In response to price discrimination by platforms, consumers may employ identity management. Recent literature on pricing discrimination related to consumer identity management has emerged, although it predominantly adopts a static perspective that solely examines the initial decisions of platforms. The relevant research related to this paper mainly involves BBPD, tailored pricing, and price discrimination under consumer identity management.

Price discrimination has historically been a subject of discourse in economics, and it pertains to the "ratchet effect". The so-called ratchet effect typically refers to the fact that firms set higher pieces for consumers who exhibit a larger willingness to pay. The research on price discrimination initially focused on the static perspective. A large number of literatures adopt the Hotelling model to study the competition of firms' third-degree price discrimination[10]. Shaffer and Zhang consider a static model with exogenously given segmented target markets, in which firms compete using coupons[10]. They derive that the conclusion is the prisoner's dilemma. Subsequent researches are gradually expanded to the dynamic perspective, which takes into account of consumer information collection in the model, and this framework is called behavior-based price discrimination (BBPD).

The first researches related to this paper is BBPD, and there are a large number of researches[4,6,11-18]. They explore the dynamic pricing of firms with BBPD. Under this framework, the competition among firms is repetitive. After attracting consumers to make their first purchase through competition, firms will infer consumers' preferences from their past purchases. Then they set corresponding prices for different consumers based on the information. When consumers tend to choose a particular firm in the first stage, the firm will realize that these consumers are its loyal consumers and their willingness to pay for its products is higher than that of other consumers who did not choose it. As a result, the firm will set higher prices for these loyal consumers and lower prices for the loyal consumers of other firms. Villas-Boas and Fudenberg and Tirole lay the foundation for the research on BBPD[12-13].

The literatures have not achieved consensus about the welfare implications of BBPD. One perspective posits that, relative to the scenario in which firms refrain from price discrimination, BBPD will exacerbate competition and adversely affect firms' profitability, unless there exists a certain degree of heterogeneity at either the firm or consumer level, in which case the outcome may be reversed[11-13,15,17]. The literature explains this view from two dimensions. The first is from the perspective of switching costs[11,17]. When consumers switch to another firm in the second stage, they will face switching costs. The existence of these costs enables firms to practice price discrimination. At this time, firms can set lower prices for consumers in the first stage to lock them in and then raise prices in the second stage. Compared with the case of no price discrimination, due to the intensified competition in the first stage, firms' profits will still be impaired. The other is from the perspective of consumers' brand preferences[13]. Firms can obtain preference information through consumers' consumption behaviors, so as to perform price discrimination on consumers. At this time, compared with the case of no price discrimination, the competition between firms will be intensified, and their profits will decline.

Another perspective states that profits of firms increase in the BBPD. First, firms can make BBPD profitable by providing enhanced service for loyalty consumers[8,15]. Second, when there are vertical differences in products, BBPD benefits low-quality firm[19-20]. Third, consumer-level heterogeneity (consumer loyalty) can also benefit BBPD. This is because BBPD can help firms filter out price-sensitive consumers[16]. Finally, consumers' concern for fairness can also benefit BBPD. This is because consumers' concern for fairness weakens the competition in the second stage, and the competition in the second stage will, in turn, weaken the competition in the first stage, ultimately leading to an increase in profits[13,21].

Additionally, Carroni incorporates BBPD into the research of platforms[22]. He considers the price discrimination on the platform, and the conclusions is consistent with those on the one-sided platform. Chen and Gong introduce the pricing strategy of BBPD into two-sided markets and analyze the pricing strategy of BBPD for the On-Demand Service Platform[5]. They examine how the third-degree price discrimination based on behavior affect platform profits, consumer surplus, and provider surplus when there exists network externality. The difference between this paper and aforementioned researcher lies in the fact that platforms implement personalized pricing rather than third-degree price discrimination. Moreover, we consider the consumer identity management and explore the effect of consumer identity management on BBPD of platforms.

In conclusion, although there is a large amount of literature on BBPD, no literature has yet introduced consumer identity management into the BBPD of platforms. The difference between this paper and the aforementioned literature is that we introduce ex post identity management into BBPD of platforms, and investigate the impact of ex post identity management on BBPD of platforms.

The second category of researches related to this paper focuses on the personalized pricing of platforms. In recent years, with the development of the platform economy and big data, platforms collect more detailed consumer information, enabling them to set individual prices for consumers. Compared with the case of third-degree price discrimination, there is relatively less literature on personalized pricing. Researches are carried out from both static perspectives and dynamic perspectives[23-30]. The general conclusion drawn from the above literature is that personalized pricing will intensify competition and do harm to platforms compared with the case of third-degree price discrimination. This is because platform will be more aggressive in pricing if they collect more consumer information. When platforms engage in personalized pricing competition, they can set prices for each consumer. This will do harm to platforms' profits since the personalized pricing gives platforms a sense that they can protect their existing consumers more easily, making the competition among platforms more intense than in situations where there is less information. These conclusions hold when consumer identity management behavior is not considered. Once the consumer identity management is introduced into the personalized pricing, the conclusions may be reversed[31]. A research highly relevant to this paper is by Choe et al.[30], he constructs a two-period Hotelling dynamic model. In the first period, platforms conduct standard Hotelling competition, and in the second period, they use personalized pricing to compete for repeat consumers. He explores how the personalized prices based on consumer information affects the prices and profits of platforms over the two periods. Since platforms collect consumer information by consumers' purchases in the first period, platforms know its past consumers better than its competitors, which leads to the information asymmetry in the second period. The information asymmetry allows platforms to set higher personalized prices for their existing consumers in the second period. In contrast, competitors who are unaware of these consumers' specific preferences can only set a uniform price. When product differentiation is exogenously given, the information asymmetry arising in the first period and the personalized pricing implemented in the second period will lead to an asymmetric equilibrium. In this equilibrium, one platform will price more aggressively in the first period to gain a larger market share. The platform with a smaller market share will poach some consumers in the second period and make up for the loss of market share in the first period with a higher poaching price. However, the result of this paper is a symmetric equilibrium. The reason for this different result is that the consumer identity management makes it impossible for platforms to set different prices for new and existing consumers separately in the second period. This will increase the poaching cost for platforms. As a result, the platform with a smaller market share cannot make up for the loss of market share in the first period by poaching consumers and charging a higher poaching price. Therefore, it will price more aggressively in the first period, and ultimately, two platforms will set the same price, reaching a symmetric equilibrium. This paper extends the previous researches by considering consumer identity management.

The third research relevant to this paper is about price discrimination with consumer identity management. Faced with the situation that platforms collect a large amount of information for price discrimination, consumers will take the initiative to prevent platforms from gathering information or hide their identities. The literature on consumer identity management mainly focuses on ex-post and in-process identity management, and mainly considers the situation of monopoly enterprises. Taylor[7], Villas-Boas[14], Acquisti and Varian are the first to conduct research in this area[8], examining the case of ex-post identity management. They find that consumer identity management will make the situation of monopoly enterprises worse. Once consumers anticipate that they may face price discrimination in the future, they will abandon their current purchases, thus preventing themselves from being identified as repeat consumers, in order to make a purchase with a lower price in the future. Consumers' strategic "waiting" will decrease firms' profits by reducing their sales volume and weakening their price discrimination. In contrast, in-process identity management will improve the situation of firms[9,32]. This is because in-process identity management gives monopolists the right to promise not to engage in price discrimination in repeat purchases. The difference between this paper and the above researches lies in the consideration of ex-post identity management and the situation of competitive platforms.

Regarding ex-post identity management, Montes et al. construct a model in which at least one of the duopoly firms can use consumers' private information for price discrimination[33], and consumers can conduct identity management by paying a "privacy cost". They study the impacts of price discrimination on prices, profits, and consumer surplus. The difference from this paper is that consumer information is sold by a monopoly data broker in that paper. In the equilibrium, only one firm can obtain consumer information, and it is a static model. However, we introduce a dynamic model that consumer information is obtained by platforms through competition in the first stage and all platforms can acquire consumers information in equilibrium. Chen et al. study the impacts of consumers' ex-post identity management on the price strategies of competitive enterprises and the surpluses of various participants from a static perspective[31]. Each competitive firm is exogenously given a segmented target market, and firms have complete information about consumers of their target markets and can carry out personalized pricing for them. Consumers can bypass firms' price discrimination through identity management. They analyze the impacts of consumer identity management when the target markets of the two firms overlap, do not fully cover each other, or exactly cover each other. The results show that the competition will be intensified since the consumer information enables firms to protect their target markets better when there do not exist consumer identity management. However, when consumers conduct identity management, the competition will be weakened since the cost for firms to serve non-target consumers increases. Therefore, firms' profits may increase in the consumer identity management, but consumer surplus and social welfare may decrease in the consumer identity management. We extend the above mode to a dynamic situation, evaluate whether platforms have incentives to compete for consumers if they expect consumers to conduct identity management, and verify whether the conclusions of Chen et al. still hold in this paper[31].

Laussel introduces consumer identity management into the two-period BBPD in a one-sided market[34]. The results show that consumer identity management increase firms' profits in the second period. However, due to fierce competition in the first period, when consumers are myopic or firms are sufficiently patient, it may lead to a decline in the total profits of firms over the two periods. The difference between our research and the above paper are network externalities. We explore the impact of consumer identity management on platforms' BBPD in a two-sided market and derive new conclusions regarding the impact of network externalities on consumer identity management and platforms' BBPD.

Overall, although there is a large amount of literature on BBPD, researches on personalized pricing competition and consumer identity management generally focus on one-sided markets. There are few researches that incorporate the consumer identity management into the BBPD of two-sided platforms.

3 MODEL SETTINGS AND RESULTS

3.1 Model Settings

Suppose there are two competitive platforms, A and B, which provide ride-hailing services in the market, such as Didi and Shouqi car-hailing platforms. Each platform connects two-sided users-consumers and service providers (drivers). Service providers provide services with horizontal differentiation, and the platform has no influence on the service differentiation. The differences of services provided by the two platforms are reflected in the Hotelling model. Assume that platform A is located at position 0 of the line segment, and platform B is located at position 1 of the line segment. All costs of platforms are standardized to 0. Consumers are uniformly distributed on the Hotelling line segment, and their quantity is standardized to 1, with a demand for one unit of service per period. Consumers located at position x are simply referred to as consumer x. It is assumed that the number of service providers is large enough and the costs they incur when providing services is extremely low, so they can meet the service demands of consumers. The reason for such an assumption is that the cost for drivers to join a platform is extremely low in reality, and they are relatively flexible in providing services. Therefore, there are a sufficient number of drivers in the online car-hailing platforms. Moreover, more and more drivers start to use electric vehicles to serve consumers with very low per-kilometer costs, so they can meet the service demands of consumers.

Platforms set service prices for consumers, while service providers can determine other elements. For instance, drivers have ownership of vehicles used to serve consumers, which are usually their private vehicles. The service price p_i set by platform i for consumers can be regarded as the final price consumers need to pay after receiving the service, or as a price ratio, such as the per-kilometer taxi fare (the travel distance of consumers is standardized as 1). It is assumed that consumers are single-homing, meaning they only join one platform per period and conduct transactions. The reason for this assumption is that when platforms develop to a certain extent, consumers may develop a preference for a particular platform, or they may dislike downloading multiple ride-hailing apps. Due to the existence of indirect network externalities, the utility of consumers is affected by the number of service providers on the platform. The more service providers there are, the greater the utility consumers will obtain. Let αn_i^S be the total indirect network externality benefits that consumers obtain from interacting with service providers on platform $i\in\{A,B\}$, where $\alpha\in(0,1)$ represents the unit network externality benefits of consumers, and n_i^S is the number of providers on platform i. Therefore, if the utility of the service for consumers is $v\geq 0$, and the unit transportation cost for consumers to any platform is $t\geq 0$, the surplus that consumer x obtains from purchasing one unit of service from platform A is $u_A=v-p_A-tx+\alpha n_A^S$, the surplus that consumer x obtains from purchasing one unit of service from platform B is $u_B=v-p_B-t(1-x)+\alpha n_B^S$. It is assumed that v is large enough so that all consumers will purchase one unit of service per period.

Service providers are not required to pay membership fees when registering to join platforms, but they need to pay the platform a proportion $(1-\lambda)$ of the transaction fee for each transaction, where $\lambda\in(0,1)$. When each transaction is finished, the service provider obtains a revenue λp_i based on the service price p_i. In practice, the cost for drivers to join platforms like Didi or Shouqi is extremely low, and they are required to pay a transaction fee to the platform for each transaction, so this assumption is in line with the real situation. Since service providers do not need to pay some costs when registering to join a platform, service providers will register to join two or more platforms simultaneously in order to have more access to consumers, i.e., service providers are multi-homing. Although service providers join two platforms at the same time and have access to all consumers, they are unable to serve consumers of two platforms at the same time. Therefore, whether a service provider joins a platform depends on the total number of consumers on the platform, and the final decision of service providers depends on the platform's pricing level. Motivated by Chen and Gong[5], the number of service providers who join and serve platform i in the equilibrium is given by

$$n_i^S=\beta\left(n_A^C+n_B^C\right)+\lambda p_i \tag{1}$$

Where $\beta\left(n_A^C+n_B^C\right)$ represents the total benefits of indirect network externality for service providers, and λp_i represents the benefits from serving consumers.

Since the total number of consumers is standardized to 1, we have $n_A^C+n_B^C=1$. Therefore, the equilibrium number of service providers of the platform i is

$$n_i^S=\beta+\lambda p_i \tag{2}$$

The game has two periods, denoted as $\tau=1$、2. In period $\tau=1a$, neither of the two platforms collect consumer information. They conduct standard Hotelling price competition simultaneously, and set prices p_{1A} and p_{1B} respectively. In period $\tau=1b$, consumers make decisions on purchasing services, and service providers provide services. After consumers purchase services in period $\tau=1b$, platforms will use tools such as "cookies" to track consumers' preference information. Therefore, at the end of period $\tau=1$, platforms can not only know whether a certain consumer purchases services from their own platforms but also obtain all the preference information of their consumers, i.e., the specific positions on the Hotelling line segment. Let A be the set of consumers who purchase services from platform A in period $\tau=1$; and let $\mathscr{B}$ be the set of consumers who purchase services from platform B in period $\tau=1$.

In period $\tau=2$, platforms set two types of prices based on the consumer information obtained in period $\tau=1$. In period $\tau=2a$, two platforms simultaneously set a uniform price for new consumers who do not choose their own platforms in period $\tau=1$. Platform A sets a uniform price p_{2AU} for consumers in set $\mathscr{B}$, and platform B sets a uniform price p_{2BU} for consumers in set A. All consumers can observe these two uniform prices, that is, the prices set for new consumers by two platforms are common knowledge. In period $\tau=2b$, two platforms conduct personalized pricing for old consumers who choose their own platforms in period τ=1. Platform A sets a personalized price $p_{2A}(x)$ for each consumer $x \in$ A, and platform B sets a personalized price $p_{2B}(y)$ for each consumer $y \in \mathscr{B}$. The personalized prices set by platforms can only be observed by individual consumers. In period τ=2c, consumers make purchases of services after observing the uniform prices and corresponding personalized prices, and then service providers provide services. The timing of the game is shown in Figure 1. The equilibrium in this paper is a subgame perfect Nash equilibrium. It is assumed that consumers only care about prices, and consumer information is only used for service pricing. Following the pricing sequence in previous literature, platforms first set uniform prices and then set personalized prices. This sequence indicates that uniform pricing requires decisions from the higher management level, and thus its adjustment speed is slower than that of personalized pricing.

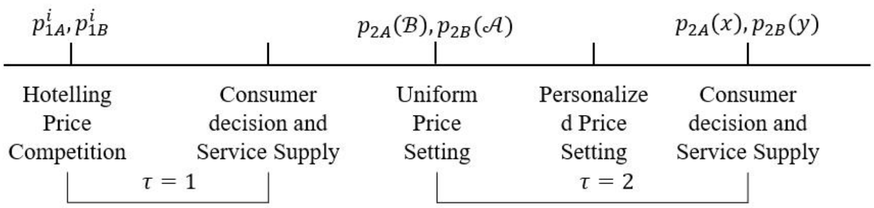

Figure 1 The Timing of the Game

Since the model includes a stage where platforms collect information from interactions with consumers, it is a dynamic model. When platforms and consumers make decisions in period τ=1, platforms discount the profits in period τ=2 at a rate of $\delta_f \in [0,1]$, and consumers discount the utilities in period τ=2 at a rate of $\delta_c \in [0,1]$. The decision-making problem for platform i in period τ=1 is to maximize the two-period profits, that is $\Pi_i = \pi_{1i} + \delta_f \pi_{2i}$; the decision-making problem for consumer x in period τ=1 is to maximize the two-period utilities, that is $U_x = u_{1x} + \delta_c u_{2x}$. When $\delta_c = 0$, it indicates that consumers are myopic and only consider the current-period surplus when making decisions in period τ=1. As δ_c increases, consumers will place more importance on the future. Since the service cost of service providers is extremely low and service providers are multi-homing with low switching costs between two platforms, it is assumed that service providers are myopic and only consider the current-period surplus when making decisions.

Although the uniform prices set by platforms in period τ=2a can be observed by all consumers, platforms may set up some obstacles to prevent old consumers who purchased from them in period τ=1 from choosing the uniform prices they have set. However, some consumers can bypass these obstacles through identity management and choose the lower uniform price. Therefore, it is assumed that there are two types of consumers. One type is active consumers. These consumers can conduct identity management, bypass the obstacles set by the platform, and choose among three prices. Active consumers $x \in$ A can choose from the uniform price of platform A, the personalized price, and the uniform price of platform B, i.e., $\{p_{2A}(x), p_{2AU}, p_{2BU}\}$. Similarly, active consumers $y \in \mathscr{B}$ can choose among $\{p_{2B}(y), p_{2BU}, p_{2AU}\}$.

The other consumer type is passive consumers. These consumers are unable to conduct identity management, so they cannot choose the uniform price set by the platform in period τ=1. For example, passive consumers $x \in$ A can only choose between the personalized price set by platform A and the uniform price set by platform B, that is, $\{p_{2A}(x), p_{2BU}\}$. Similarly, passive consumers $y \in \mathscr{B}$ can only choose between $\{p_{2B}(y), p_{2AU}\}$. Since the ex-post identity management is closer to the real situation, this paper only considers the case of ex-post identity management.

3.2 Benchmark Results

For subsequent reference, this subsection presents the benchmark results of BBPD of platforms in the absence of the consumer identity management. In this scenario, all consumers do not conduct the identity management. In the second period, platforms can price separately for old consumers and new consumers. That is, they can set personalized prices for old consumers and uniform prices for new consumers.

Let the marginal consumer z be the consumer who is indifferent between purchasing from platform A and platform B in period τ=1. By Choe et al. and Chen and Gong[5,30], we obtain the equilibrium of BBPD for platforms without the consumer identity management as follows.

Proposition 1. (Equilibrium without consumer identity management) When no consumers conduct identity management, the BBPD of platforms results in two asymmetric equilibria, and there exists unidirectional consumer switching in the second period. The platform with a larger market share will lose some consumers but still retain a relatively large market share. The equilibrium is given by

(1) In the first period, two platforms set prices $p_{1A}^{\mathrm{p}} = \frac{t(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$ and $p_{1B}^{\mathrm{p}} = \frac{t\left[6\delta_c^2+3\delta_c(-8+\delta_f)-2\left(-12+3\delta_f+\delta_f^2\right)\right]}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$. The marginal consumer is located at $z^{\mathrm{p}} = \frac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)} \leq \frac{1}{2}$. The number of service providers are $n_A^S = \beta + \frac{\lambda t(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$ and $n_B^S = \beta + \frac{\lambda t\left[6\delta_c^2+3\delta_c(-8+\delta_f)-2\left(-12+3\delta_f+\delta_f^2\right)\right]}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$.

In the second period, the prices of two platforms are

$$p_{2A}^{p}(x)=\begin{cases}\frac{t(1-2x)}{(1-\alpha\lambda)}, & \text{if } x\in\left[0,\frac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)}\right]\\ \frac{\delta_f t}{(1-\alpha\lambda)(12-6\delta_c+\delta_f)}, & \text{if } x\in\left[\frac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)},1\right]\end{cases} \quad (3)$$

$$p_{2B}^{p}(y)=\begin{cases}0, & \text{if } y\in\left[0,\frac{3(2-\delta_c)}{12-6\delta_c+\delta_f}\right]\\ t\left[\frac{2y}{(1-\alpha\lambda)}-\frac{6(2-\delta_c)}{(12-6\delta_c+\delta_f)(1-\alpha\lambda)}\right], & \text{if } y\in\left[\frac{3(2-\delta_c)}{12-6\delta_c+\delta_f},1\right]\end{cases} \quad (4)$$

(2) In the first period, two platforms set prices $p_{1A}^{p}=\frac{t\left[6\delta_c^2+3\delta_c(-8+\delta_f)-2\left(-12+3\delta_f+\delta_f^2\right)\right]}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$ and $p_{1B}^{p}=\frac{t(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$, the marginal consumer is located at $z^{p}=\frac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)}\geq\frac{1}{2}$. The number of service providers are $n_A^S=\beta+\frac{\lambda t\left[6\delta_c^2+3\delta_c(-8+\delta_f)-2\left(-12+3\delta_f+\delta_f^2\right)\right]}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$ and $n_B^S=\beta+\frac{\lambda t(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$.

In the second period, the prices of two platforms are

$$p_{2A}^{p}(x)=\begin{cases}t\left[\frac{2(6-3\delta_c+\delta_f)}{(12-6\delta_c+\delta_f)(1-\alpha\lambda)}-\frac{2x}{(1-\alpha\lambda)}\right], & \text{if } x\in\left[0,\frac{6-3\delta_c+\delta_f}{12-6\delta_c+\delta_f}\right]\\ 0, & \text{if } x\in\left[\frac{6-3\delta_c+\delta_f}{12-6\delta_c+\delta_f},1\right]\end{cases} \quad (5)$$

$$p_{2B}^{p}(y)=\begin{cases}\frac{\delta_f t}{(1-\alpha\lambda)(12-6\delta_c+\delta_f)}, & \text{if } y\in\left[0,\frac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)}\right]\\ \frac{t(2y-1)}{(1-\alpha\lambda)}, & \text{if } y\in\left[\frac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)},1\right]\end{cases} \quad (6)$$

Where the superscript "p" represents the scenario where all consumers do not conduct identity management, that is, the case where consumers are all passive consumers.

From Proposition 1 we have that two platforms do not necessarily share the market equally in the first period when all consumers do not conduct identity management. The marginal consumer may be located at either $z^{p}\leq 1/2$ or $z^{p}\geq 1/2$. The number of service providers on two platforms is also different, and the prices of services for two platforms in two periods are not symmetric. When platforms obtain consumer information in the first period and implement personalized pricing for old consumers and uniform pricing for new consumers in the second period, they can price new and old consumers separately since consumers do not conduct identity management. This leads to asymmetric price-cutting incentives for two platforms in the first period. Specifically, a more aggressive platform can lower its price in the first period to gain a larger market share and then charge higher personalized prices to all old consumers in the second period. Therefore, the more aggressive platform benefits from a larger market share and higher personalized prices. For the less aggressive platform, although it loses a part of its market share, it can poach some consumers in the second period and charge a higher poaching price to make up for the loss of market share in the first period. As a result, if one platform has a stronger incentive to low prices to compete for market share than the other platform, and finally two platforms will reach an asymmetric equilibrium.

4 EQUILIBRIUM OF PLATFORMS' BBPD WITH CONSUMER IDENTITY MANAGEMENT

This section discusses the scenario where all consumers conduct identity management and there is no cost for identity management. At this time, all consumers can bypass the obstacles set by the platform without any cost. Besides the personalized prices set by the platform, they can also choose from two uniform prices. This leads to two possible price strategies for the platform in the second period. One is to serve only old consumers with personalized prices, and the other is to serve all consumers with a lower uniform price. The reason is that old consumers of the platform are more willing to purchase from the platform than new consumers, that is, they are more loyal to the platform and are willing to pay higher prices, resulting in the higher prices for old consumers than those for new consumers.

When the platform wants to implement higher personalized pricing for old consumers and lower uniform pricing for new consumers, it is very difficult for the platform to implement personalized pricing since old consumers can choose the lower uniform price of the platform by identity management which do not need any cost. Therefore, if the platform wants to make the personalized pricing effective, it must set a high enough uniform price so that no consumer will accept it.

Let the marginal consumer z still be the consumer who is indifferent between purchasing from platform A and platform B in period τ=1. We use backward induction to solve this game. The process is as follows: First, given z, we derive the equilibrium prices in period τ=2. Second, given the equilibrium in period τ=2, we obtain the equilibrium prices and z in period τ=1.

Lemma 1 presents the equilibrium of period τ = 2, which is shown in Figure 2.

Lemma 1. (The equilibrium of period τ = 2) When all consumers conduct identity management and there is no cost for identity management, there are three equilibria in the second period:

(1) There exists an equilibrium of perfect price discrimination (PPD) if and only if $z\in\left[\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t},\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right]$. Each platform sets a sufficiently high uniform price $p_{2AU},p_{2BU}\geq\frac{v+\alpha\beta}{1-\alpha\lambda}$ in

period τ=2a, so that all consumers will not choose this price but rather the personalized prices set in period τ=2b. In period τ=2b, each platform sets the highest personalized price to extract all the consumer surplus from the old consumers.

(1) Platform A sets the personalized price $p_{2A}(x)=\frac{v-tx+\alpha\beta}{1-\alpha\lambda}$ for consumers $x\in A=[0,z]$, and platform B sets the personalized price $p_{2B}(y)=\frac{v-t(1-y)+\alpha\beta}{1-\alpha\lambda}$ for consumers $y\in \mathscr{B}=[z,1]$. Since each platform sets the highest personalized price and implements the perfect price discrimination, this equilibrium is called the PPD equilibrium. There is no consumer switching in the equilibrium.

(2) The one-way poaching equilibrium of platform A occurs if and only if $z\in\left[0,\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right)$. In period τ=2a, platform A sets a uniform price $p_{2AU}=\frac{t}{2(1-\alpha\lambda)}$ to serve consumers in the interval $\left[0,\frac{1}{4}\right]$. Platform B sets a sufficiently high uniform price $p_{2BU}\geq\frac{3t}{2(1-\alpha\lambda)}$ in period τ=2a to prevent its existing consumers from choosing this price, and then sets a personalized price $p_{2B}(y)=\frac{t(4y-1)}{2(1-\alpha\lambda)}$ in period τ=2b to serve consumers in the interval $\left[\frac{1}{4},1\right]$. In the equilibrium, there is a one-way conversion of consumers from platform B to platform A.

(3) The one-way poaching equilibrium of platform B occurs if and only if $z\in\left(\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t},1\right]$. Platform A sets a sufficiently high uniform price $p_{2AU}\geq\frac{3t}{2(1-\alpha\lambda)}$ in period τ=2a and sets a personalized price $p_{2A}(x)=\frac{t(3-4x)}{2(1-\alpha\lambda)}$ in period τ=2b to serve consumers in the interval $\left[0,\frac{3}{4}\right]$. Platform B sets a uniform price $p_{2BU}=\frac{t}{2(1-\alpha\lambda)}$ in period τ=2a to serve consumers in the interval $\left[\frac{3}{4},1\right]$. In the equilibrium, there is a one-way conversion of consumers from platform A to platform B.

Proof. See Appendix 1.

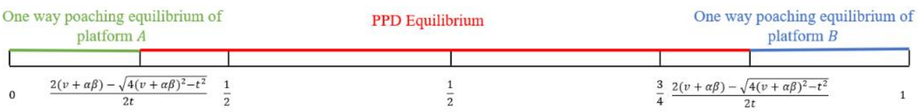

Figure 2 Equilibrium in Period τ = 2

It can be seen that from Lemma 1 that there are three equilibria in the second period. When each platform obtains a relatively large market share in the first period, i.e., the marginal consumer in the first period is in the interval $z\in\left[\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t},\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right]$, each platform set a sufficiently high uniform price in the second period so that no consumer accept it, and they set the highest personalized prices for their respective old consumers to extract the surplus of all consumers, which leads to the PPD equilibrium. The economic intuition is that when both platforms have relatively large market shares in the second period, if a platform sets a lower uniform price to poach consumers from its competitor, its existing old consumers choose this lower uniform price through identity management.

Therefore, when both platforms have large market shares, neither platform sets a lower uniform price to poach consumers from its competitor. Instead, they naturally reach a tacit understanding to serve their own old consumers at PPD prices, ultimately leading to the PPD equilibrium.

When the market share of a platform is a sufficiently small in the first period, that is, $z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}$ (the market share of platform *A* is sufficiently small) or $z>\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}$ (the market share of platform *B* is sufficiently small), the platform with the smaller market share sets a lower uniform price to poach some consumers from the platform with the larger market share, and serve both its own old consumers and the poached consumers at this uniform price, leading to the deviation from the PPD equilibrium in the second period. The economic intuition behind this is twofold. On the one hand, although the platform with the smaller market share deviates from the PPD equilibrium by setting a lower uniform price and can no longer implement perfect price discrimination against its old consumers, the loss of its profits is also relatively small due to its small market share.

On the other hand, the lower uniform price of the platform enables it to poach some consumers from its competitor, thus its profits will increase due to the increase of the market share. When the market share of a platform enterprise is small enough, the effect of increased market share on profits outweighs the effect of the inability to implement perfect price discrimination on profit, making it profitable for the platform to deviate from the PPD equilibrium. At this time, although the platform with the larger market share loses some consumers, its market share remains relatively large. Thus, it is profitable for the platform to implement a lower personalized price, which is lower than the PPD price, is still more advantageous than implementing a uniform price. Therefore, two platforms will eventually reach a one-way poaching equilibrium, either the one-way poaching equilibrium of platform *A* or that of platform *B*.

In addition, when v is sufficiently large (the requirement that the marginal consumer surplus is larger than 0, $v+\alpha\beta>3t/2$), we have $\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}<\frac{2v-\sqrt{4v^2-t^2}}{2t}$ and $\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}>\frac{-2(v+\alpha\beta-t)+\sqrt{4v^2-t^2}}{2t}$. This shows that, compared with the one-sided market without network externalities, in the two-sided market, platforms are more willing to implement perfect price discrimination during the stage of using consumer information with the existence of consumer identity management, and all the consumer surplus is extracted at this stage. Since $\frac{\partial\left[\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right]}{\partial\alpha}<0$, $\frac{\partial\left[\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right]}{\partial\beta}<0$, $\frac{\partial\left[\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right]}{\partial\alpha}>0$ and $\frac{\partial\left[\frac{-2(v+\alpha\beta-t)+\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\right]}{\partial\beta}>0$, we derive that the interval in which two platforms implement perfect price discrimination in the second period becomes larger with the increase of the indirect network externalities. That is, both platforms will implement perfect price discrimination and reach the PPD equilibrium within a broader range of market shares. The above conclusions can be elaborated by the impact of network externalities on consumer identity management and platforms' BBPD in Proposition 2.

Proposition 2. (The impact of network externalities on consumer identity management and BBPD) When consumers conduct identity management with no cost, platforms in the two-sided market are more willing to implement perfect price discrimination to extract all the consumer surplus with the consumer identity management during collecting consumer information due to the existence of network externalities compared with the one-sided market. As network externalities increase, the willingness of platforms to implement perfect price discrimination is strengthened.

The economic intuition behind Proposition 2 is that, due to the existence of indirect network externalities in the two-sided market, consumers obtain additional network externality benefits, which are unavailable in the one-sided market. After consumers gain these benefits, the PPD prices set by platforms in the second period which extract all consumer surplus are higher than those in the one-sided market. That is to say, for platform A, $\frac{v-tx+\alpha\beta}{1-\alpha\lambda}>v-tx$, and for platform B, $\frac{v-t(1-y)+\alpha\beta}{1-\alpha\lambda}>v-t(1-y)$. Since platforms set higher PPD prices and acquire larger profits in the two-sided market in the second period, they will still implement perfect price discrimination and reach the PPD equilibrium even with a smaller market share. This indicates that consumer identity management enables platforms to reach the PPD equilibrium within a broader range of market shares in the two-sided market compared to the one-sided market due to the existence of network externalities. That is, they prefer to implement perfect price discrimination. Moreover, benefits of the indirect network externality for consumers increase with network externalities, which leads to the higher prices of platforms and perfect price discrimination in the second period.

Based on the equilibrium of the second period, we derive the second-period profits of Platform A and Platform B as follows

$$\pi_{2A}=\begin{cases}\frac{t(1-\lambda)}{8(1-\alpha\lambda)}, & \text{if } 0\leq z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\\ \frac{(1-\lambda)}{1-\alpha\lambda}\left[(v+\alpha\beta)z-\frac{1}{2}tz^2\right], & \text{if } \frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\leq z\leq\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}\\ \frac{9t(1-\lambda)}{16(1-\alpha\lambda)}, & \text{if } \frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}<z\leq 1\end{cases} \tag{7}$$

$$\pi_{2B}=\begin{cases}\frac{9t(1-\lambda)}{16(1-\alpha\lambda)}, & \text{if } 0\leq z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\\ \frac{(1-\lambda)}{1-\alpha\lambda}\left[v+\alpha\beta-\frac{1}{2}t-z(v+\alpha\beta-t)-\frac{1}{2}tz^2\right], & \text{if } \frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\leq z\leq\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}\\ \frac{t(1-\lambda)}{8(1-\alpha\lambda)}, & \text{if } \frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}<z\leq 1\end{cases} \tag{8}$$

We derive the marginal consumer z by the equilibrium of the second period and profits of two platforms. Lemma 2 shows that the expression of the marginal consumer z when consumer identity management exists and the relationship between z and δ_c (the degree to which consumers value the future).

Lemma 2. (The relationship between the marginal consumer z and δ_c)

When all consumers conduct identity management with no cost, the marginal consumer who is indifferent between purchasing products from two platforms in the first period is located at $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$. The surplus of the marginal consumer in the second period does not affect the choice in the first period. That is, consumers make decisions in the first period only the current-period surplus.

We can explain Lemma 2 from consumers and platforms. First, since consumers can conduct identity management with no cost in the second period to conceal their information, they can thwart the price discrimination practices of platforms. They can not only choose the personalized prices set by platforms for old consumers but also the uniform prices set for new consumers. Therefore, consumers only need to consider the surplus of the current period when making choices in the first period. The other is from the perspective of platforms. When all consumers conduct identity management with no cost, platforms can only choose one pricing strategy in the second period, i.e., personalized pricing or uniform pricing. Specifically, two platforms implement perfect price discrimination in the second period under the PPD equilibrium and the surplus of consumers in the second period is zero. Therefore, when making decisions in the first period, the marginal consumer only needs to consider the surplus of the current period, that is, the utility obtained from the two platforms is the same, i.e., $v-tz-p_{1A}+\alpha(\beta+\lambda p_{1A})=v-t(1-z)-p_{1B}+\alpha(\beta+\lambda p_{1B})$. Then we have $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$. By Lemma 1 we derive that the marginal consumer z is indifferent between purchasing from platform A in both periods and purchasing from platform B in the first period and switching to platform A in the second period under the one-way poaching equilibrium of platform A, that is $v-tz-p_{1A}+\alpha(\beta+\lambda p_{1A})+\delta_c\left[v-tz-p_{2AU}+\alpha\left(\beta+\lambda p_{2A}(z)\right)\right]=v-t(1-z)-p_{1B}+\alpha(\beta+\lambda p_{1B})+\delta_c[v-tz-p_{2AU}+\alpha(\beta+\lambda p_{2AU})]$. Then we have $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$. The economic intuition is that platform A is unable to price separately for new and old consumers in the second period since consumers conduct identity management with no cost. At this time, the marginal consumer pays the same uniform price whether they purchase from platform A in the second period or switch from platform B to platform A. Therefore, the marginal consumer only needs to consider the surplus of the current period when making decisions in the first period. Finally, the solution for the marginal consumer z under the one-way poaching equilibrium of platform B is similar to that of platform A. Lemma 2 indicates that δ_c does not affect the position of the marginal consumer z.

From the expression of the marginal consumer z in period $\tau=1$, we obtain the equilibrium price of $\tau=1$. platforms will discount the profits of period $\tau=2$ when making decisions in period $\tau=1$. By the marginal consumer z and the profits in period $\tau=2$, we derive the two-period discounted profits Π_A and Π_B of two platforms.

$$\Pi_A=\begin{cases}\frac{p_{1A}(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{9t(1-\lambda)}{16(1-\alpha\lambda)},\ \text{if} \\ p_{1B}-\frac{t}{1-\alpha\lambda}\le p_{1A}<p_{1B}-\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{1-\alpha\lambda} \\ \frac{(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]\{\delta_f[4(v+\alpha\beta)-t-(1-\alpha\lambda)p_{1B}]+(4+\delta_f)(1-\alpha\lambda)p_{1A}\}}{8t(1-\alpha\lambda)},\text{if} \\ p_{1B}-\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)}\le p_{1A}\le p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)} \\ \frac{p_{1A}(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{t(1-\lambda)}{8(1-\alpha\lambda)},\ \text{if} \\ p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}<p_{1A}\le p_{1B}+\frac{t}{(1-\alpha\lambda)}\end{cases} \tag{9}$$

$$\Pi_B=\begin{cases}\frac{(1-\lambda)p_{1B}[t-(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{9t(1-\lambda)}{16(1-\alpha\lambda)},\ \text{if} \\ p_{1A}-\frac{t}{1-\alpha\lambda}\le p_{1B}<p_{1A}-\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{1-\alpha\lambda} \\ \frac{(1-\lambda)[t+(1-\alpha\lambda)(p_{1A}-p_{1B})]\{\delta_f[4(v+\alpha\beta)-t-(1-\alpha\lambda)p_{1A}]+(4+\delta_f)(1-\alpha\lambda)p_{1B}\}}{8t(1-\alpha\lambda)},\text{if} \\ p_{1A}-\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}\le p_{1B}\le p_{1A}+\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)} \\ \frac{(1-\lambda)p_{1B}[t-(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{t(1-\lambda)}{8(1-\alpha\lambda)},\ \text{if} \\ p_{1A}+\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{1-\alpha\lambda}<p_{1B}\le p_{1A}+\frac{t}{1-\alpha\lambda}\end{cases} \tag{10}$$

By optimizing the two-period discounted profits of two platforms, we obtain the equilibrium of $\tau=1$ in Proposition 3. The superscript "a" in Proposition 3 represents the situation where all consumers conduct identity management, that is, the case where consumers are all active consumers.

Proposition 3. (Equilibrium of $\tau=1$) When all consumers conduct identity management with no cost, two platforms set symmetric prices $p_{1A}^a=p_{1B}^a=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$ in equilibrium in the first period. The marginal consumer is located at $z^a=\frac{1}{2}$, and δ_c does not affect the price of platforms in the first period. The number of service providers is $n_A^S=n_B^S=\beta+\lambda\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$.

Proof. See Appendix 2.

From Proposition 3, two platforms set the same price $p_{1A}^a=p_{1B}^a=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$ in the first period. Two platforms split the market share evenly, and the number of service providers on both platforms is identical. The price set by platforms in the first period moves in the opposite direction to the utility v, δ_f, and the unit network externality benefit of service providers is β. The reason is as follows: As v increases, the consumer surplus that the platform extracts in the second period increases. Thus, platforms compete more vigorously for consumers in the first period and set lower prices. As δ_f becomes larger, the platform attaches more importance to the future, then the competition in the first period will be more intense, and the price in the first period will also be lower. As β increases, service providers gain more network externality benefits, so they are more willing to join the platform and serve consumers. That is to say, the number of service providers on the platform will increase, and the benefits of network externality of consumers will also increase. This enables platforms to set higher prices in the second period, leading the competition in the first period even fiercer and lowing prices. From Lemma 2, we derive that consumers can hide their information by the identity management in the second period. Therefore, they only need to consider the current-period surplus when making decisions in the first period, which means that the price of platforms in the first period is independent of δ_c. In addition, since the surplus of the marginal consumer should be larger than or equal to 0, it is required that $v-\frac{1}{2}t-\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}+\alpha\left(\beta+\lambda\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}\right)\geq 0$, then we have that $v+\alpha\beta\geq\frac{t(3+\delta_f)}{2(1+\delta_f)}$. Given $\delta_f\in[0,1]$, $\frac{t(3+\delta_f)}{2(1+\delta_f)}$ decreases in δ_f. Thus, we derive $v+\alpha\beta\geq\frac{3}{2}t$. This indicates that the utility of consumers plus the product of the benefits of the indirect network externality of consumers and service providers $\alpha\beta$, should be greater than or equal to $\frac{3}{2}$ times the unit transportation cost t.

By the equilibria of the first period and the second period, we derive the equilibrium of platforms' BBPD.

Proposition 4. (the equilibrium with the consumer identity management) When all consumers conduct identity management with no cost, platforms' BBPD leads to a unique symmetric equilibrium. The equilibrium results of the two-period dynamic game are as follows

(1) In the first period, two platforms set the same price $p_{1A}^a=p_{1B}^a=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$, and evenly divide the market share, i.e., the marginal consumer is located at $z^a=\frac{1}{2}$. The number of service providers is $n_A^S=n_B^S=\beta+\lambda\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$. Each platform's profit is$\pi_{1A}^a=\pi_{1B}^a=\frac{(1-\lambda)\{2t+\delta_f[t-2(v+\alpha\beta)]\}}{4(1-\alpha\lambda)}$, and the consumer surplus is $CS_1^a=v+\alpha\beta+\delta_f\left(v+\alpha\beta-\frac{t}{2}\right)-\frac{5t}{4}$.

(2) In the second period, two platforms implement perfect price discrimination and reach the PPD equilibrium. At this time, two platforms first set relatively high uniform prices $p_{2AU},p_{2BU}\geq\frac{v+\alpha\beta}{1-\alpha\lambda}$, which are unacceptable to consumers. Subsequently, platform A serves consumers who is location at $x\in\left[0,\frac{1}{2}\right]$ at the personalized price $p_{2A}(x)=\frac{v-tx+\alpha\beta}{1-\alpha\lambda}$, and platform B serves consumers who is location at $y\in\left[\frac{1}{2},1\right]$ at the personalized price $p_{2B}(y)=\frac{v-t(1-y)+\alpha\beta}{1-\alpha\lambda}$. In the equilibrium, there is no consumer switching. Each platform obtains the same profit $\pi_{2A}^a=\pi_{2B}^a=\frac{(1-\lambda)[4(v+\alpha\beta)-t]}{8(1-\alpha\lambda)}$, and the consumer surplus is $CS_2^a=0$.

Proposition 4 shows that consumer identity management enables platforms to reach a unique symmetric equilibrium in BBPD. Two platforms set the same price in the first period to evenly divide the market and implement perfect price discrimination against consumers in the second period, squeezing out all of the consumers' surplus. In the second period, neither of two platforms has the incentive to deviate from the PPD equilibrium, nor they will poach consumers from their competitors. There is no consumer switching in the equilibrium. From Proposition 1, however, if consumers do not conduct identity management, platforms' BBPD leads to two asymmetric equilibria.

The main reason for the difference in equilibrium symmetry lies in whether platforms can price new and old consumers separately in the second period. When all consumers do not conduct identity management, platforms can price new and old consumers separately in the second period. They set higher personalized prices for old consumers and lower uniform prices for new consumers. At this time, platforms have asymmetric incentives to low prices in the first period. The more aggressive platform can gain a larger market share in the first period by setting a lower price and then set high personalized prices for old consumers in the second period. For the less aggressive platform, although it obtains a smaller market share in the first period, it can not only set personalized prices for old consumers but also poach some of its competitors' consumers in the second period and charge a higher poaching price, thus making up for the loss of market share. Therefore, two platforms have different motives to weaken their competitors, resulting in an asymmetric equilibrium. However, when consumers conduct identity management, the less aggressive platform cannot price new and old consumers separately in the second period. When poaching competitors' consumers, it cannot implement effective personalized pricing for old consumers, which increases the cost of poaching competitors' consumers and thus weakens the platform's motivation to poach consumers.

Platforms cannot make up for the loss of market share in the first period through higher poaching prices. As a result, they will compete more fiercely for consumers in the first period. Eventually, two platforms have the same incentive to cut prices, leading to a symmetric equilibrium with an equal share of the market. In addition, the high PPD profits of platforms in the second period due to consumer the identity management lead platforms to low prices in the first period. Finally, they split the market share equally and reach a symmetric equilibrium.

By comparing the equilibrium prices in Proposition 4 and that in the one-sided market, we obtain that when consumers conduct identity management with no cost, network externalities intensify the competition of the platform in the first period and lead to lower prices in most cases, i.e., $\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}<\frac{2t+\delta_f(t-2v)}{2}$. Only when the benefit of the unit indirect network externality are limited ($0<\alpha<\frac{-t\lambda+\delta_f+t\lambda\delta_f}{\lambda\delta_f}, 0<\beta<\frac{-t\lambda+t\lambda\delta_f}{-\delta_f+\alpha\lambda\delta_f}$) and the utility of the service is also limited ($\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{2t\lambda-2\beta\delta_f+t\lambda\delta_f}{2\lambda\delta_f}$)), the price increases in the network externalities. The reason is that when the benefits of the network externality and the utility of services are low, the PPD profits of platforms in the second period are also low, which leads to less competition and higher prices in the first period. When consumers conduct identity management with no cost, the price increases in the network externalities.

Comparative static analysis. Since we consider a two-period dynamic model, it is necessary to conduct a comparative static analysis of the discount factor. When all consumers conduct identity management with no cost, consumers only consider the surplus of the current period when making decisions. Therefore, δ_c does not affect the equilibrium. When $t=1$, $v=4$, $\alpha=0.5$, $\beta=0.5$ and $\lambda=0.8$, the comparative static analysis of the discount factor of platforms δ_f is shown in Figure 3.

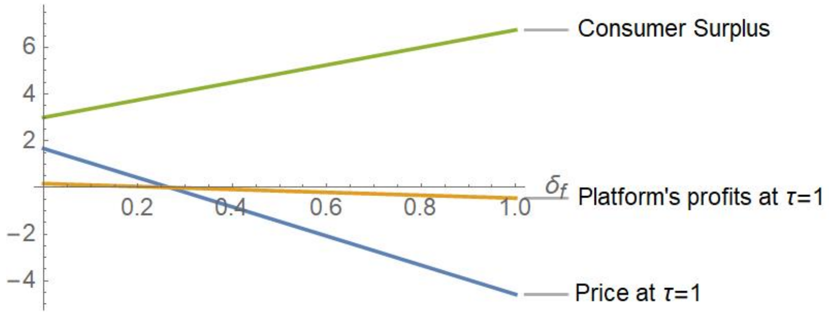

Figure 3 The Relationship between the Equilibrium Price of the First Period, Platform Profits, and Consumer Surplus and δ_f

From Figure 3, as δ_f increases, platforms attach more importance to the future, and the competition in the first period becomes more intense. Therefore, the price and the profits of platforms in the first period will decline, while the consumer surplus will increase.

5 COMPARISON OF THE EQUILIBRIA WITH AND WITHOUT THE IDENTITY MANAGEMENT

This section compares the equilibrium results of platforms' BBPD with and without consumer identity management. Specifically, we compare the price of platforms in the two periods, platform profits, consumer surplus, and social welfare. Furthermore, we explore how consumers' ex-post identity management affects platforms' BBPD, and whether platforms will still be motivated to compete for consumers, collect information, and implement BBPD if they anticipate that consumers will conduct identity management.

5.1 The Comparison of Prices

From Section 4, we obtain that the BBPD of platforms is affected by the consumer identity management. When consumers conduct the identity management in the second period, platforms cannot price new and old consumers separately, thus platforms reach a symmetric equilibrium by setting the same price in the first period. However, from Proposition 1, when consumers do not conduct the identity management, platforms can set personalized prices for old consumers while set a uniform price for new consumers in the second period. Moreover, the platform with a smaller market share will poach some consumers from the platform with a larger market share, ultimately reaching an asymmetric equilibrium. Therefore, whether consumers conduct the identity management will affect the prices. It is necessary to compare the equilibrium prices in the two situations to evaluate the price effect of the consumer identity management.

From Proposition 1 and Proposition 4, the equilibrium price with the consumer identity management in the first period is $p_1^{\mathrm{a}}=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$, and the equilibrium price of the platform with a smaller market share and a larger market share without the consumer identity management in the first period are $p_{1,s}^{\mathrm{p}}=\frac{t(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$ and $p_{1,\ b}^{\mathrm{p}}=\frac{t\left[6\delta_c^2+3\delta_c(-8+\delta_f)-2\left(-12+3\delta_f+\delta_f^2\right)\right]}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$. Where the subscript "s" (small) represents the platform with a smaller market share, and the subscript "b" (big) represents the platform with a larger market share. By Mathematica, we derive the prices in Proposition 5.

Proposition 5. (The prices with the consumer identity management) The consumer identity management will intensify the competition between platforms in the first period, and the price will decrease ($\mathrm{p}_1^{\mathrm{a}}<\mathrm{p}_1^{\mathrm{p}}$). This indicates that platforms still have incentives to compete for consumers although platforms know that consumers may conduct identity management. Only when platforms are myopia and the utility of services is limited, the competition between platforms in the first period is relieved and prices may increase ($\mathrm{p}_2^{\mathrm{a}}>\mathrm{p}_2^{\mathrm{p}}$).

Proposition 5 shows that the consumer identity management will intensify the competition between two platforms in the first period and the price will decrease. Only when platforms are myopia and the utility of services is limited (for the platform with a smaller market share, the conditions are $0<\delta_f<\frac{-12\delta_c+6\delta_c^2}{-20+11\delta_c}$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{12t\delta_c-6t\delta_c^2+16t\delta_f-24\alpha\beta\delta_f-7t\delta_c\delta_f+12\alpha\beta\delta_c\delta_f+3t\delta_f^2-2\alpha\beta\delta_f^2}{24\delta_f-12\delta_c\delta_f+2\delta_f^2}$; for the platform with a larger market share, the conditions are $0<\delta_f<\frac{-12\delta_c+6\delta_c^2}{-16+9\delta_c}$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{12t\delta_c-6t\delta_c^2+20t\delta_f-24\alpha\beta\delta_f-9t\delta_c\delta_f+12\alpha\beta\delta_c\delta_f+3t\delta_f^2-2\alpha\beta\delta_f^2}{24\delta_f-12\delta_c\delta_f+2\delta_f^2})$), the results will be reversed.

The economic intuition behind the comparison of prices of the first-period can be explained by the transmission mechanism of platforms' profits. When consumers do not conduct the identity management, platforms price new and old consumers separately in the second period. Platforms charge a higher poaching price to make up for the loss of market share, then the competition between platforms is relieved and the price in the first period will increase. However, when consumers conduct the identity management, platforms cannot price new and old consumers separately in the second period. The less aggressive platform cannot make up for the loss of market share in the first period through a higher poaching price, then the competition will be intensified for consumers and prices will be lower in the first period. In addition, the consumer identity management enables platforms to obtain high PPD profits in the second period. The PPD profits increases in the utility of each service. This also drives platforms to compete more fiercely for consumers and set lower prices in the first period, even though they know that consumers will conduct identity management. Therefore, compared with the scenario without the identity management, when platforms value the high profits in the second period and the utility of each service is relatively high, the prices in the first-period decrease in the consumer identity management.

In most cases, the price of platforms decreases in the consumer identity management. Only when platforms are myopia and the utility of services is low, i.e., when the PPD profits are low and the profit mechanism of platforms is weak, prices in the first period increases in the consumer identity management.

When consumers conduct the identity management, platforms cannot price new and old consumers separately in the second period. Regarding the economic intuition behind the comparison results of the second-period prices, as analyzed previously, when consumers conduct identity management, platforms cannot price new and old consumers separately in the second period. As a result, the cost of poaching competitors' consumers increases, which makes both platforms refrain from poaching each other's consumers. Instead, they spontaneously set the highest personalized prices to squeeze out all of the remaining consumer surplus when serving their own old consumers, leading to the PPD equilibrium. At this point, there is no competition for consumers, and the prices are highest.

However, when consumers do not conduct the identity management, platforms will price new and old consumers separately in the second period. The platform with a smaller market share in the first period will poach some consumers from the platform with a larger market share in the second period, leading to a one-way poaching equilibrium. At this time, there exists relatively intense competition for consumers, and the prices are lower. Therefore, compared with the situation without the identity management, the consumer identity management will weaken the competition between platforms in the second period and drive prices up.

5.2 The Comparison of Profits

From Section 4, we derive that the BBPD of platforms is affected by the consumer identity management. When consumers conduct the identity management in the second period, platforms can no longer price new and old consumers separately, then two platforms will reach the PPD equilibrium and set the same price in the first period, which leads to a symmetric equilibrium. However, when consumers do not conduct the identity management, platforms set personalized prices for old consumers while setting a uniform price for new consumers in the second period by Proposition 1. Moreover, the platform with a smaller market share will poach some consumers from the platform with a larger market share, leading to an asymmetric equilibrium. Therefore, whether consumers conduct the identity management cause platforms to set different prices, it is necessary to compare the equilibrium prices in the two scenarios to evaluate the price effect of consumer identity management.

From Proposition 1 and 4, the equilibrium price with the consumer identity management is $p_1^{a}=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$ in the first period, while the equilibrium price of the platform with a smaller market share is $p_{1,s}^{p}=\frac{t(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$, and the equilibrium price of the platform with a larger market share is $p_{1,\ b}^{p}=\frac{t\left[6\delta_c^2+3\delta_c(-8+\delta_f)-2\left(-12+3\delta_f+\delta_f^2\right)\right]}{2(1-\alpha\lambda)(12-6\delta_c+\delta_f)}$without the consumer identity management. Where the subscript "s" (small) represents the platform with a smaller market share, and the subscript "b" (big) represents the platform with a larger market share. By Mathematica, we can obtain Proposition 5.

Proposition 6. (The price effect of the consumer identity management) The consumer identity management will intensify the competition among platform enterprises in the first period, and the price will decrease ($p_1^a<p_1^p$). This indicates that although platforms know that consumers will conduct the identity management, they still have incentives to compete for consumers. Only when platforms are myopia and the utility of services is limited, the competition between platforms in the first period will weaken and prices will increase ($p_1^a>p_1^p$). The consumer identity management will weaken the competition between platforms in the second period, and prices will increase ($p_2^a>p_2^p$).

Proposition 5 indicates that the consumer identity management will intensify the competition between platforms in the first period, and prices will be lower in most cases. Only when platforms are myopia and the utility of service is also limited (for platforms with a smaller market share, the conditions are $0<\delta_f<\frac{-12\delta_c+6\delta_c^2}{-20+11\delta_c}$ and $\frac{1}{2}(3t-2\alpha\beta)\le v<\frac{12t\delta_c-6t\delta_c^2+16t\delta_f-24\alpha\beta\delta_f-7t\delta_c\delta_f+12\alpha\beta\delta_c\delta_f+3t\delta_f^2-2\alpha\beta\delta_f^2}{24\delta_f-12\delta_c\delta_f+2\delta_f^2}$; for platforms with a larger market share, the conditions are $0<\delta_f<\frac{-12\delta_c+6\delta_c^2}{-16+9\delta_c}$ and $\frac{1}{2}(3t-2\alpha\beta)\le v<\frac{12t\delta_c-6t\delta_c^2+20t\delta_f-24\alpha\beta\delta_f-9t\delta_c\delta_f+12\alpha\beta\delta_c\delta_f+3t\delta_f^2-2\alpha\beta\delta_f^2}{24\delta_f-12\delta_c\delta_f+2\delta_f^2}$)), the results will be reversed.

We can explain the comparison results of the first-period prices by the transmission mechanism of platforms' profits. When consumers do not engage in the identity management, platforms can price new and old consumers separately in the second period. This enables less aggressive platforms to recoup their loss of market share by charging higher poaching prices, leading to weaker competition and relatively higher prices in the first period. However, when consumers conduct the identity management, platforms cannot price new and old consumers separately in the second period. As a result, less aggressive platforms cannot make up for the loss of market share in the first period by higher poaching prices. Thus, they will compete more fiercely for consumers and set lower prices in the first period. Moreover, the consumer identity management allows platforms to obtain high PPD profits in the second period. The PPD profits increase in the utility of each service. This also motivates platforms to compete more vigorously for consumers and set even lower prices in the first period, even though they anticipate that consumers will conduct identity management. Finally, compared with the situation without the identity management, when platforms value the high profits in the second period of and the utility of each service is relatively high, the prices of the first period will decrease in the consumer identity management.

In most cases, the prices in the first period will decrease in the consumer identity management. Only when platforms are myopia and the utility of services is low, i.e., the PPD profits are low and the profit mechanism of platforms is weak, the prices will increase in the consumer identity management.

We can explain the comparison results of the second-period prices as follows: When consumers conduct the identity management, platforms cannot price new and old consumers separately in the second period. The cost of poaching consumers from competitors increases, then neither of platforms will poach the other's consumers from its competitors. Instead, they spontaneously set the highest personalized prices to extract all the remaining consumer surplus from their own old consumers, reaching the PPD equilibrium. At this time, there is no competition for consumers, and prices are highest. However, when consumers do not conduct the identity management, platforms can price new and old consumers separately in the second period. The platform with a smaller market share in the first period will poach some consumers from the platform with a larger market share in the second period, leading to a one-way poaching equilibrium. At this time, there exists competition for consumers, the competition is relatively intense, and prices are lower. Therefore, the consumer identity management will weaken the competition between two platforms in the second period and drive the prices up.

5.3 The Comparison of Consumer Surplus and Social Welfare

When consumers conduct the identity management, the consumer surplus in the first period is $CS_1^a=v+\alpha\beta+\delta_f\left(v+\alpha\beta-\frac{t}{2}\right)-\frac{5t}{4}$, the consumer surplus in the second period is $CS_2^a=0$, and the total consumer surplus over the two periods is $CS^a=v+\alpha\beta+\delta_f\left(v+\alpha\beta-\frac{t}{2}\right)-\frac{5t}{4}$.

When consumers do not conduct the identity management, the consumer surplus in the first period is $CS_1^p=\frac{1}{4(12-6\delta_c+\delta_f)^2}\left\{\begin{matrix}144[-5t+4(v+\alpha\beta)]+72t\delta_c^3+24\delta_f[t+4(v+\alpha\beta)]+\delta_f^2[59t+4(v+\alpha\beta)]+4t\delta_f^3+\\12\delta_c^2[-39t+12(v+\alpha\beta)+t\delta_f]-4\delta_c[-36(7t-4(v+\alpha\beta))+3\delta_f(3t+4(v+\alpha\beta))+8t\delta_f^2]\end{matrix}\right\}$, the consumer surplus in the second period is $CS_2^p=\frac{36t-48(v+\alpha\beta)-6(3t-4(v+\alpha\beta))\delta_c+(5t-4(v+\alpha\beta))\delta_f}{24\delta_c-4(12+\delta_f)}$, and the total consumer surplus over the two periods is $CS^p=\frac{1}{2(12-6\delta_c+\delta_f)^2}\left\{\begin{matrix}-576(t-v-\alpha\beta)+36t\delta_c^3+\delta_f[-36t+96(v+\alpha\beta)]+\delta_f^2[27t+4(v+\alpha\beta)]+2t\delta_f^3+\\6\delta_c^2[24(-2t+v+\alpha\beta)+t\delta_f]-2\delta_c[-72(5t-4(v+\alpha\beta))-3(t-8(v+\alpha\beta))\delta_f+8t\delta_f^2]\end{matrix}\right\}$.

By comparing the consumer surplus in two scenarios, we can obtain Proposition 7. Proposition 7 also compares social welfare in two scenarios. Since each consumer always purchases one unit of service per period, the comparison of social welfare can be transformed into the comparison of consumers' transportation costs.

Proposition 7. The consumer surplus increases in the consumer identity management in the first period in most cases. Only when platforms are myopia and the utility of service is also limited, the results will be reversed. The consumer surplus in the second period and the total consumer surplus over the two periods decrease in the consumer identity management, while social welfare increases in the consumer identity management.

From Proposition 7, we derive that the consumer surplus increases in the consumer identity management in the first period. By Proposition 5, we obtain that platforms will lower prices in the first period, which benefits consumers. However, when platforms are myopia and the utility of service is also limited, platforms will increase prices in the first period, which leads to the decrease of the consumer surplus. In the second period, the consumer identity management enables platforms to extract all of the consumer surplus, which leads to the decrease of consumer surplus. When consumers do not conduct the identity management, platforms can price new and old consumers separately in the

second period. As a result, the personalized prices set for old consumers by both platforms will be restricted by the poaching prices of competing platforms, which leads the lower prices and higher consumer surplus. On the contrary, when consumers conduct the identity management, the consumer surplus in the second period is zero. Therefore, the consumer surplus in the second period decreases in the consumer identity management. Although the consumer surplus in the first period increases in the consumer identity management in most cases, the value of the increase is less than the value of decrease of the consumer surplus in the second period. Therefore, the total consumer surplus over the two periods decreases in the consumer identity management. When consumers conduct the identity management, the marginal consumer is located at $z^{a}=1/2$. On the contrary, when consumers do not conduct the identity management, the marginal consumer is located at $z^{p}\leq 1/2$ or $z^{p}\geq 1/2$. Therefore, the consumer identity management will shorten the average traveling distance of consumers and thus social welfare will increase.
The comparison of the equilibrium under the two scenarios is be summarized in Table 1.

Table 1 The Comparison of the Equilibrium

	τ=1	τ=2	Total
Price	For platforms with a smaller market share in most cases: $p_1^a<p_1^p$. Only when $0<\delta_f<\frac{-12\delta_c+6\delta_c^2}{-20+11\delta_c}$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{12t\delta_c-6t\delta_c^2+16t\delta_f-24\alpha\beta\delta_f-7t\delta_c\delta_f+12\alpha\beta\delta_c\delta_f+3t\delta_f^2-2\alpha\beta\delta_f^2}{24\delta_f-12\delta_c\delta_f+2\delta_f^2}$ we have $p_1^a>p_1^p$。 For platforms with a larger market share in most cases: $p_1^a<p_1^p$. Only when $0<\delta_f<\frac{-12\delta_c+6\delta_c^2}{-16+9\delta_c}$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{12t\delta_c-6t\delta_c^2+20t\delta_f-24\alpha\beta\delta_f-9t\delta_c\delta_f+12\alpha\beta\delta_c\delta_f+3t\delta_f^2-2\alpha\beta\delta_f^2}{24\delta_f-12\delta_c\delta_f+2\delta_f^2}$, we have $p_1^a>p_1^p$.	$p_2^a>p_2^p$	
Profits of platforms	For platforms with a smaller market share in most cases: $\pi_1^a<\pi_{1,s}^p$. Only when $0<\delta_f<\mathrm{Root}[4\#1^3+\#1^2(24-13\delta_c)-144\delta_c+144\delta_c^2-36\delta_c^3+\#1(192-216\delta_c+60\delta_c^2)\&,1]$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{1}{2\delta_f(12-6\delta_c+\delta_f)^2}\{36t\delta_c^3+24\delta_c^2[-6t+\delta_f(2t-3\alpha\beta)]-\delta_f[48(-5t+6\alpha\beta)-48\delta_f(t-\alpha\beta)$ we have $\pi_1^a>\pi_{1,s}^p$. For platforms with a larger market share in most cases: $\pi_1^a<\pi_{1,b}^p$. Only when $0<\delta_f<\mathrm{Root}[4\#1^3+144\delta_c-144\delta_c^2+36\delta_c^3+\#1^2(-4+3\delta_c)+\#1(-240+264\delta_c-72\delta_c^2)\&,2]$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{1}{2\delta_f(12-6\delta_c+\delta_f)^2}\{36t\delta_c^3+36\delta_c^2[-4t+\delta_f(t-2\alpha\beta)]-3\delta_c(-48t+8(7t-12\alpha\beta)\delta_f+(11$, we have $\pi_1^a>\pi_{1,b}^p$.	$\pi_2^a>\pi_2^p$	For platforms with a smaller market share: $\Pi^a>\Pi_s^p$ For platforms with a larger market share: if $0.077<\delta_c\leq 1$, $\Pi^a>\Pi_b^p$.
Consumer surplus	In most cases: $CS_1^a>CS_1^p$。 Only when $0<\delta_f<\frac{3}{8}\left(-18+11\delta_c+\sqrt{324-332\delta_c+89\delta_c^2}\right)$ and $\frac{1}{2}(3t-2\alpha\beta)\leq v<\frac{1}{2\delta_f(12-6\delta_c+\delta_f)^2}\{36t\delta_c^3+6\delta_f\delta_c^2[-24t+(7t-12\alpha\beta)]-4\delta_c[-36t+24\delta_f(2t-3\alpha\beta)+\delta$, $CS_1^a<CS_1^p$.	$CS_2^a<CS_2^p$	$CS^a<CS^p$
Social welfare	$W_1^a>W_1^p$	$W_2^a>W_2^p$	$W^a>W^p$

6 EXTENSIONS

This section extends the model in Section 3 to analyze the situation where there are costs associated with consumer identity management. A new stage is added to the original two-period (τ=2) game process. In period τ=2a, two platforms simultaneously set a uniform price for new consumers. In period τ=2b, two platforms simultaneously set personalized prices for old consumers. In period τ=2c, consumers observe the uniform and personalized prices set by the platforms and then decide whether to conduct the identity management. If they do, they need to pay an additional cost c on top of the original service price. In other words, if the service price is p_i, the price ultimately paid by consumers who choose identity management is p_i+c. In period τ=2d, consumers make purchasing decisions, and service providers offer services. Since when $c>v$, no consumers will conduct the identity management, it is assumed that $0<c\leq v$.
The model remains a dynamic sequential game and is solved using the backward induction. First, given the marginal consumer z, we derive the equilibrium prices in period τ=2. Secondly, given the equilibrium in period τ=2, we obtain the equilibrium prices and z in period τ=1. The solution approach is similar to the scenario where all consumers can conduct the identity management with no cost. If consumers conduct the identity management with no cost, the only difference is that when platforms deviate from the PPD equilibrium and set a relatively low uniform price, some old consumers who are closer to the platform (or more loyal to the platform) will choose the uniform price by the identity management

since the personalized price set for them by the platform is higher. Moreover, old consumers who are farther from the platform may continue to choose the personalized price since the personalized price set by the platform might be lower than the sum of the uniform price and the identity management cost. Proposition 8 presents the equilibrium of BBPD of platforms in the case where there exist costs of the consumer identity management.

Proposition 8. When the cost of the consumer identity management belongs to $0<c\leq v$, if only a part of consumers conduct the identity management, the second-period PPD equilibrium occurs if and only if $z\in\left[\frac{1}{2}+\frac{[v+\alpha\beta-c(1-\alpha\lambda)](1-\sqrt{2})}{t},\frac{1}{2}-\frac{[v+\alpha\beta-c(1-\alpha\lambda)](1-\sqrt{2})}{t}\right]$; If all consumers conduct the identity management, the second-period PPD equilibrium occurs if and only if $z\in\left[\frac{2[v+\alpha\beta-c(1-\alpha\lambda)]-\sqrt{4[v+\alpha\beta-c(1-\alpha\lambda)]^2-t^2}}{2t},\frac{-2(v+\alpha\beta-t-c(1-\alpha\lambda))+\sqrt{4[v+\alpha\beta-c(1-\alpha\lambda)]^2-t^2}}{2t}\right]$, and both platforms implement perfect price discrimination in the second period. As c becomes smaller, then the range of z that can obtain the PPD equilibrium will be wider.

Proof. See Appendix 3.

Proposition 8 shows that although there are costs of the consumer identity management, there exists an interval of $z(c)$ within which two platforms still implement perfect price discrimination and reach the PPD equilibrium in the second period. As c decreases, the interval of $z(c)$ will expand.

Therefore, when the cost of the consumer identity management is lower than a certain threshold, even though there exist costs of the identity management, the equilibrium without the cost of the identity management can still be obtained. This shows that the conclusions in this paper are robust, and the smaller the value of *c*, the more robust the conclusions are.

7 CONCLUSIONS AND DISCUSSIONS

7.1 Applications in Consumer Privacy Policies

The advent of the era of big data, algorithmic recommendation, and personalized pricing make people pay more and more attention to the protection of consumer privacy. On June 29, 2018, the state of California in the United States passes the California Consumer Privacy Act. China also passes the Personal Information Protection Law of the People's Republic of China on August 20, 2021. Consumer privacy protection policies set limits in terms of rules for the collection, storage, sale, and disclosure of consumer information, and also grant consumers various rights such as the right to know, the right to be forgotten, and the right of access.

When consumers are granted more rights, they can have more comprehensive control over their own information. If these consumer rights lead more and more consumers to engage in the identity management, it will harm consumers' interests. As shown in the analysis of this paper, when consumers conduct the identity management, it will weaken the competition among platforms in the second period and enable two platforms to implement perfect price discrimination, thus the consumer surplus decrease. Therefore, if policymakers formulate consumer privacy policies from the perspective of consumer interests, they should remind consumers that the active identity management behavior, which seemingly benefits consumers, may not actually improve consumer welfare and may even harm it.

However, the analysis in this paper also indicates that active identity management by consumers can improve social welfare. Thus, if policymakers formulate consumer privacy policies based on social welfare, they should include provisions that can encourage consumers to engage in the identity management. This shows that whether policies should promote consumers to conduct active identity management depends on whether policymakers place more importance on consumer welfare or social welfare.

7.2 Applications in the Management of Platforms

The general conclusion of this paper is that platforms still have incentives to compete for consumers and obtain consumer information even if they expect consumers to conduct the identity management. For platforms with a smaller market share, the platform's profits increase in the consumer identity management. For platforms with a larger market share, when consumers are farsighted, the platform's profits increase in the consumer identity management. By the conclusions of this paper, we can derive some suggestions for the consumer information management and pricing strategies of platforms.

First of all, in the face of increasingly common consumer identity management behaviors such as hiding identity information and using price comparison software to seek low prices for new consumers, platforms should not think that competing for consumers and collecting consumer information for price discrimination are no longer in their own interests. Although consumer identity management intensifies the competition among platforms in the first period of collecting consumer information, it weakens the competition in the second period of using consumer information and allows platforms to implement stronger price discrimination and obtain high profits, ultimately benefiting the platforms. Therefore, when a large number of consumers conduct the identity management, it is still in the interests of the platforms to compete fiercely for consumers and obtain personal consumer information for price discrimination.

Secondly, although strict privacy policies will pose many obstacles for platforms when collecting consumer information and seemingly go against the interests of the platforms, if strict privacy policies grant consumers more rights and enable more consumers to conduct active identity management, the profits of platforms with a smaller market share will increase in these strict privacy policies. Moreover, if consumers value future consumption behaviors, the profits of platforms with a larger market share will increase. Therefore, platforms can grant consumers more rights, collect and use information more transparently, and give consumers sufficient control over their own information in practice, so that consumers can conduct the identity management more actively.

7.3 Conclusion

With the development of big data and the platform economy, platforms can easily track consumers, obtain consumer information, and use it for price discrimination. Faced with price discrimination by platforms, consumers often try to thwart such attempts by deleting cookies, creating new accounts for repeated purchases, which means consumers are engaging in the identity management. Therefore, it is crucial to evaluate whether the BBPD of platforms are affected by consumers' identity management behaviors. Additionally, it is necessary to determine whether platforms still have incentives to compete for consumers, collect consumer information, and implement BBPD if they know consumers to engage in the identity management. It is also essential to investigate the impact of network externalities on consumers' identity management and platforms' BBPD.

This paper addresses these problems by constructing a two-period dynamic duopoly model with horizontal differentiation. In this model, platforms engage in standard Hotelling price competition in the first period and personalized pricing competition for repeat-purchasing old consumers and uniform pricing competition for new consumers in the second period. Besides purchasing services each period, consumers conduct the identity management after observing the prices set by platforms for repeat purchases in the second period. After consumers making their decisions, service providers offer services. This paper reaches the following conclusions:

(1) The BBPD of platforms are affected by consumers' identity management behaviors. The consumer identity management usually intensifies competition during the stage when platform enterprises collect consumer information, leading to price declines. Only when both the condition that platform enterprises place limited importance on the future and the condition that the utility that services can bring to consumers is limited are satisfied simultaneously will the situation be reversed. Consumer identity management weakens competition during the stage when platform enterprises use consumer information, enabling platform enterprises to implement perfect price discrimination, extract all consumer surplus, and drive prices up.

(2) The consumer surplus decreases in the consumer identity management, but social welfare increases in the consumer identity management. Moreover, when consumers are farsighted, it is also beneficial to all platforms.

(3) Even if platforms know that consumers will engage in the identity management, they still have incentives to compete for consumers, collect consumer information, and implement BBPD.

(4) The consumer identity management makes platform enterprises more willing to implement perfect price discrimination during the stage of using consumer information, and this willingness strengthens as network externalities increase in two-sided markets compared with one-sided markets.

(5) When the cost of the identity management is less than a threshold, the above conclusions hold. Various consumer privacy protection policies which are introduced for platforms' BBPD may not always be beneficial to consumers. If such policies prompt consumers to engage in the identity management, they will do harm to consumers' interests.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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APPENDIX

Appendix 1: The proof of Lemma 1

For the equilibrium of $\tau=2$, since all consumers engage in the identity management with no cost, platforms cannot price separately for new and old consumers and can only choose one form between personalized prices and uniform prices. Therefore, we first analyze the existence of the PPD equilibrium in which both platforms set a uniformly high price that is not chosen by any consumers and serve with effectively personalized prices. Then we analyze whether there is an equilibrium that deviates from the PPD, that is, an equilibrium in which a certain platform sets a lower uniform price to poach competitors' consumers.

(1) Proof of the existence of the PPD equilibrium

Case 1: $\mathbf{0}\leq z\leq\frac{\mathbf{1}}{\mathbf{2}}$

First, we analyze whether platform A deviate from the PPD equilibrium. Suppose platform A deviates from the PPD equilibrium by setting $p_{2AU}^{d}<\frac{v+\alpha\beta}{1-\alpha\lambda}$. The interval $\left[\frac{1}{2},1\right]$ is the non-competitive interval of platform B. The so-called non-competitive interval means that for any poaching price $p_{2AU}^{d}\geq0$ set by platform A, platform B always use a personalized price $p_{2B}(y)\geq0$ to attract consumer y to choose itself, that is, the interval where competitors cannot poach. Faced with the poaching price p_{2AU}^{d} set by platform A with the intention of poaching platform B's consumers, consumers $y\in[z,1]$ of platform B will choose between $p_{2B}(y)$ and p_{2AU}^{d}. Consumer y chooses platform B if and only if $v-p_{2B}(y)-t(1-y)+\alpha n_{B}^{S}\geq v-p_{2AU}^{d}-ty+\alpha n_{A}^{S}$. Since $n_{B}^{S}=\beta+\lambda p_{2B}(y)$ and $n_{A}^{S}=\beta+\lambda p_{2AU}^{d}$, substituting it into the inequality, we get $p_{2B}(y)\leq p_{2AU}^{d}+\frac{t(2y-1)}{1-\alpha\lambda}$. This means that the optimal personalized price set by platform B for consumer y is

$p_{2B}(y)=\max\left\{p_{2AU}^{d}+\frac{t(2y-1)}{1-\alpha\lambda},0\right\}$. Since the most aggressive poaching price of platform A is $p_{2AU}^{d}=0$, platform B can protect its own consumers by relying on the personalized pricing $p_{2B}(y)=\max\left\{\frac{t(2y-1)}{1-\alpha\lambda},0\right\}$. When $\frac{t(2y-1)}{1-\alpha\lambda}=0$, $y=\frac{1}{2}$, so the maximum non-competitive interval of platform B is $\left[\frac{1}{2},1\right]$.

Therefore, platform A will deviate from the PPD equilibrium only when some consumers in the interval $\left[z,\frac{1}{2}\right]$ purchase from platform A at the price p_{2AU}^{d}. The optimal response of platform B in the interval $\left[z,\frac{1}{2}\right]$ is to set a personalized price of 0. Define the marginal consumer $\tilde{y}\in\left[z,\frac{1}{2}\right]$ as the consumer who is indifferent between purchasing from platform A and platform B, that is, $v-t\tilde{y}-p_{2AU}^{d}+\alpha(\beta+\lambda p_{2AU}^{d})=v-t(1-\tilde{y})-0+\alpha(\beta+0)$. Then we obtain $\tilde{y}=\frac{1}{2}-\frac{p_{2AU}^{d}(1-\alpha\lambda)}{2t}$. Since $\tilde{y}\in\left[z,\frac{1}{2}\right]$, we have $z\leq\tilde{y}=\frac{1}{2}-\frac{p_{2AU}^{d}(1-\alpha\lambda)}{2t}$. Therefore, $p_{2AU}^{d}\leq\frac{t(1-2z)}{1-\alpha\lambda}$. The problem of maximizing the deviation profit for platform A is given by

$$\begin{cases}\max\limits_{p_{2AU}^{d}}\pi_{2A}^{d}=\left[\frac{1}{2}-\frac{p_{2AU}^{d}(1-\alpha\lambda)}{2t}\right](1-\lambda)p_{2AU}^{d}\\ s.t.\ p_{2AU}^{d}\leq\frac{t(1-2z)}{1-\alpha\lambda}\\ p_{2AU}^{d}\geq0\end{cases}\quad(1)$$

By the Kuhn-Tucker conditions, we obtain the optimal uniform price for platform A's deviation:

$$p_{2AU}^{d}=\begin{cases}\frac{t}{2(1-\alpha\lambda)},\ \text{if } 0\leq z\leq\frac{1}{4},\\ \frac{t(1-2z)}{1-\alpha\lambda},\ \text{if } \frac{1}{4}\leq z\leq\frac{1}{2}.\end{cases}\quad(2)$$

The condition that the marginal consumer surplus is larger than 0 requires that $v+\alpha\beta\geq t$.
Then we derive the deviation profits of platform A:

$$\pi_{2A}^{d}=\begin{cases}\frac{t(1-\lambda)}{8(1-\alpha\lambda)},\ \text{if } 0\leq z\leq\frac{1}{4}\\ \frac{tz(1-2z)(1-\lambda)}{1-\alpha\lambda},\text{if } \frac{1}{4}\leq z\leq\frac{1}{2}\end{cases}\quad(3)$$

The PPD profits of platform A is $\pi_{2A}^{PPD}=\int_0^z\left[\frac{v-tx+\alpha\beta}{1-\alpha\lambda}(1-\lambda)\right]dx=\frac{(1-\lambda)}{1-\alpha\lambda}\left[(v+\alpha\beta)z-\frac{1}{2}tz^2\right]$. By comparing the PPD profit π_{2A}^{PPD} and the maximum deviation profit π_{2A}^{d}, we can find that when $\frac{2(v+\alpha\beta)-\sqrt{4[(v+\alpha\beta)]^2-t^2}}{2t}\leq z\leq\frac{1}{2}$, $\pi_{2A}^{PPD}\geq\pi_{2A}^{d}$ and platform A will not deviate from the PPD equilibrium. Only when $0\leq z<\frac{2(v+\alpha\beta)-\sqrt{4[(v+\alpha\beta)]^2-t^2}}{2t}$, platform A will deviate from the PPD equilibrium, where $\frac{2(v+\alpha\beta)-\sqrt{4[(v+\alpha\beta)]^2-t^2}}{2t}<\frac{1}{4}$.

Secondly, we analyze whether platform B will deviate from the PPD equilibrium. Based on the previous analysis, the maximum non-competitive interval of platform A is $0\leq z\leq\frac{1}{2}$. Since $0\leq z\leq\frac{1}{2}$, the interval $[0,z]$ is the non-competitive interval of platform A. Even if platform B deviates from the PPD equilibrium, it cannot poach consumers from platform A. Instead, it can only choose a uniform price p_{2BU}^{d} in the interval $[z,1]$ to maximize its profits. The PPD equilibrium enables platform B to charge the highest personalized price that extracts all consumer surplus from each consumer in the interval $[z,1]$, i.e., the perfect price discrimination. When platform B deviates from the PPD equilibrium, it can neither poach consumers from platform A nor set the highest price for each consumer. Therefore, the deviation profit of platform B must be less than the PPD profits. Finally, when $0\leq z\leq\frac{1}{2}$, platform B has no incentives to deviate from the PPD equilibrium.

Case 2: $\mathbf{\frac{1}{2}\leq z\leq1}$

First, similar to the above analysis, platform A will not deviate from the PPD equilibrium at this time.

Secondly, we analyze whether platform B will deviate from the PPD equilibrium. Assume that platform B deviates from the PPD equilibrium by setting a uniform price $p_{2BU}^{d}<\frac{v+\alpha\beta}{1-\alpha\lambda}$. Since $\left[0,\frac{1}{2}\right]$ is the non-competitive part of platform A, platform B will deviate from the PPD equilibrium only when some consumers in the interval $\left[\frac{1}{2},z\right]$ purchase from platform B at the price of p_{2BU}^{d}. In the interval $\left[\frac{1}{2},z\right]$, the optimal response of platform A is to set a personalized price of 0. Define the marginal consumer $\tilde{x}\in\left[\frac{1}{2},z\right]$ as the consumer who is indifferent between purchasing from platform A and platform B i.e., $v-t\tilde{x}-0+\alpha\beta=v-t(1-\tilde{x})-p_{2BU}^{d}+\alpha\left(\beta+\lambda p_{2BU}^{d}\right)$. Then we have $\tilde{x}=\frac{1}{2}+\frac{p_{2BU}^{d}(1-\alpha\lambda)}{2t}$. Since $\tilde{x}\in\left[\frac{1}{2},z\right]$, then $\frac{1}{2}+\frac{p_{2BU}^{d}(1-\alpha\lambda)}{2t}=\tilde{x}\leq z$. i.e., $p_{2BU}^{d}\leq\frac{t(2z-1)}{1-\alpha\lambda}$. The problem of maximizing the deviation profit for platform B is:

$$\begin{cases} \max\limits_{p_{2BU}^d} \left[1-\left(\frac{1}{2}+\frac{p_{2BU}^d(1-\alpha\lambda)}{2t}\right)\right](1-\lambda)p_{2BU}^d \\ \quad s.t.\ p_{2BU}^d \le \frac{t(2z-1)}{1-\alpha\lambda} \\ \quad\quad p_{2BU}^d \ge 0 \end{cases} \tag{4}$$

By the Kuhn-Tucker conditions to solve the optimization problem, the optimal uniform price for platform B's deviation is given by:

$$p_{2BU}^d=\begin{cases} \frac{t(2z-1)}{1-\alpha\lambda}, & \text{if } \frac{1}{2}\le z\le\frac{3}{4} \\ \frac{t}{2(1-\alpha\lambda)}, & \text{if } \frac{3}{4}\le z\le 1 \end{cases} \tag{5}$$

Then the maximum deviation profit is given by

$$\pi_{2B}^d=\begin{cases} \frac{t(2z-1)(1-z)(1-\lambda)}{1-\alpha\lambda}, & \text{if } \frac{1}{2}\le z\le\frac{3}{4} \\ \frac{t(1-\lambda)}{8(1-\alpha\lambda)}, & \text{if } \frac{3}{4}\le z\le 1 \end{cases} \tag{6}$$

The PPD profits of platform B is $\pi_{2B}^{PPD}=\int_z^1 \left[\frac{v-t(1-y)+\alpha\beta}{1-\alpha\lambda}\right](1-\lambda)\,dy=\frac{(1-\lambda)}{1-\alpha\lambda}\left[v+\alpha\beta-\frac{1}{2}t-z(v+\alpha\beta-t)-\frac{1}{2}tz^2\right]$.

By comparing π_{2B}^{PPD} and π_{2B}^d, we can obtain that when $\frac{1}{2}\le z\le\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}$, $\pi_{2B}^{PPD}\ge\pi_{2B}^d$ and platform B will not deviate from the PPD equilibrium. Only when $\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}<z\le 1$, $\pi_{2B}^{PPD}\le\pi_{2B}^d$ and platform B will deviate from the PPD equilibrium, where $\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}>\frac{3}{4}$.

Finally, combining the two cases of $0\le z\le\frac{1}{2}$ and $\frac{1}{2}\le z\le 1$, we derive that the PPD equilibrium exists if and only if $z\in\left[\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t},\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}\right]$.

(2) Proof of the existence of the unilateral poaching equilibrium of platform A

By the previous analysis, platform A will deviate from the PPD equilibrium if and only if $0\le z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}$. In this case, the optimal unified price for platform A to deviate is $p_{2AU}=\frac{t}{2(1-\alpha\lambda)}$. Next, given the unified price $p_{2AU}=\frac{t}{2(1-\alpha\lambda)}$ of platform A, we will examine the pricing strategy of platform B, that is we compare the profits of platform B under the two scenarios of implementing personalized pricing and unified pricing.

First, we consider the situation where platform B implements personalized pricing. At this time, platform B sets a relatively high unified price $p_{2B}(\text{A})>\frac{v+\alpha\beta}{1-\alpha\lambda}$ and implements personalized prices. Consumer $y\in[z,1]$ will choose platform B if and only if $v-t(1-y)-p_{2B}(y)+\alpha[\beta+\lambda p_{2B}(y)]\ge v-ty-p_{2AU}+\alpha[\beta+\lambda p_{2AU}]$. By $p_{2AU}=\frac{t}{2(1-\alpha\lambda)}$ we obtain $p_{2B}(y)\le\frac{t(4y-1)}{2(1-\alpha\lambda)}$. Define the marginal consumer $\tilde{y}$ such that $p_{2B}(\tilde{y})=\frac{t(4\tilde{y}-1)}{2(1-\alpha\lambda)}=0$. Then we derive $\tilde{y}=\frac{1}{4}$. This shows that when platform B implements the personalized pricing, platform B will serve consumers in the interval $\left[\frac{1}{4},1\right]$ with the personalized price $p_{2B}(y)=\frac{t(4y-1)}{2(1-\alpha\lambda)}$. At this time, the unified price of platform B is $p_{2BU}\ge\max\left\{\frac{t(4y-1)}{2(1-\alpha\lambda)}:y\in\left[\frac{1}{4},1\right]\right\}=\frac{3t}{2(1-\alpha\lambda)}$. The profits of platform B when implementing personalized prices is $\pi_{2B}^P=\int_{\frac{1}{4}}^1\left[\frac{t(4y-1)}{2(1-\alpha\lambda)}(1-\lambda)\right]dy=\frac{9t(1-\lambda)}{16(1-\alpha\lambda)}$.

Secondly, we consider the case where platform B implements a uniform pricing strategy. When platform B sets a uniform price p_{2BU}, define the marginal consumer $\tilde{y}\in[z,1]$ who is indifferent between choosing platform A and platform B, i.e., $v-t\tilde{y}-p_{2AU}+\alpha[\beta+\lambda p_{2AU}]=v-t(1-\tilde{y})-p_{2BU}+\alpha[\beta+\lambda p_{2BU}]$. By $p_{2AU}=\frac{t}{2(1-\alpha\lambda)}$ we have $\tilde{y}=\frac{1}{4}+\frac{p_{2BU}(1-\alpha\lambda)}{2t}$. The problem for platform B to choose p_{2BU} to maximize the uniform-pricing profit π_{2B}^U is:

$$\begin{cases} \max\limits_{p_{2BU}} \pi_{2B}^U=\left[1-\left(\frac{1}{4}+\frac{p_{2BU}(1-\alpha\lambda)}{2t}\right)\right](1-\lambda)p_{2BU} \\ \quad s.t.\ p_{2BU}\ge 0 \end{cases} \tag{7}$$

By the Kuhn-Tucker conditions, we have the optimal uniform price is $p_{2BU}=\frac{3t}{4(1-\alpha\lambda)}$. At this time, the profit of the uniform price is $\pi_{2B}^U=\frac{9t(1-\lambda)}{32(1-\alpha\lambda)}$.

Finally, by comparing the profits of the personalized price π_{2B}^P and the profit of the uniform price π_{2B}^U, we find that $\pi_{2B}^P>\pi_{2B}^U$. So when platform A deviates from the PPD equilibrium and sets a uniform price $p_{2AU}=\frac{t}{2(1-\alpha\lambda)}$, platform B will choose the personalized price $p_{2B}(y)=\frac{t(4y-1)}{2(1-\alpha\lambda)}$ to serve consumers in the interval $\left[\frac{1}{4},1\right]$, and will set a high enough uniform price $p_{2BU}\ge\frac{3t}{2(1-\alpha\lambda)}$ such that no consumer will choose this uniform price. Therefore, the unilateral poaching equilibrium of platform A exists if and only if $0\le z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}$.

(3) Proof of the existence of the unilateral poaching equilibrium of platform B

The proof of the existence of the unilateral poaching equilibrium of platform B is similar to that of platform A. The unilateral poaching equilibrium of platform B exists if and only if $\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}<z\leq 1$.

Appendix 2: Proof of Proposition 3

For the solution of the equilibrium in the first period, when two platforms make decisions in the first period, they will maximize the discounted profits of the two periods by choosing their respective prices p_{1A} and p_{1B} in the first period. Therefore, it is necessary to solve the total discounted profits of the two platforms in two periods, then we solve the price reaction functions of two platforms, and finally solve the equilibrium price of the first period by the price reaction functions.

First, we solve the total discounted profits of platform A and platform B in the two periods. By Lemma 3.1 and Appendix 1, there are three equilibria in the second period, the profits of platform A and platform B in the second period represented by the marginal consumer z is given by

$$\pi_{2A}=\begin{cases}\frac{t(1-\lambda)}{8(1-\alpha\lambda)}, & \text{if } 0\leq z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\\ \frac{(1-\lambda)}{1-\alpha\lambda}\left[(v+\alpha\beta)z-\frac{1}{2}tz^2\right], & \text{if } \frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\leq z\leq\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}\\ \frac{9t(1-\lambda)}{16(1-\alpha\lambda)}, & \text{if } \frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}<z\leq 1\end{cases} \tag{8}$$

$$\pi_{2B}=\begin{cases}\frac{9t(1-\lambda)}{16(1-\alpha\lambda)}, & \text{if } 0\leq z<\frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\\ \frac{(1-\lambda)}{1-\alpha\lambda}\left[v+\alpha\beta-\frac{1}{2}t-z(v+\alpha\beta-t)-\frac{1}{2}tz^2\right], & \text{if } \frac{2(v+\alpha\beta)-\sqrt{4(v+\alpha\beta)^2-t^2}}{2t}\leq z\leq\frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}\\ \frac{t(1-\lambda)}{8(1-\alpha\lambda)}, & \text{if } \frac{-2(v+\alpha\beta-t)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2t}<z\leq 1\end{cases} \tag{9}$$

From the profits of the second period, we derive the marginal consumer z expressed in terms of price. When making a decision in the first period, the marginal consumer z is indifferent between purchasing from platform A and platform B, i.e.,

$v-tz-p_{1A}+\alpha(\beta+\lambda p_{1A})+\delta_c\left[v-tz-p_{2A}(z)+\alpha\left(\beta+\lambda p_{2A}(z)\right)\right]=v-t(1-z)-p_{1B}+\alpha(\beta+\lambda p_{1B})+\delta_c\left[v-t(1-z)-p_{2B}(z)+\alpha\left(\beta+\lambda p_{2B}(z)\right)\right]$.

By $p_{2A}(z)=\frac{v-tz+\alpha\beta}{1-\alpha\lambda}$ and $p_{2B}(z)=\frac{v-t(1-z)+\alpha\beta}{1-\alpha\lambda}$, we get z=1/2+(1−$\alpha\lambda$)(p_1B−p_1A)/2t. The marginal consumer z is indifferent between purchasing from platform A in both periods and purchasing from platform B in the first period and then switching to platform A in the second period. That is, $v-tz-p_{1A}+\alpha(\beta+\lambda p_{1A})+\delta_c[v-tz-p_{2AU}+\alpha(\beta+\lambda p_{2AU})]=v-t(1-z)-p_{1B}+\alpha(\beta+\lambda p_{1B})+\delta_c[v-tz-p_{2AU}+\alpha(\beta+\lambda p_{2AU})]$. Then we also obtain $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$. Similarly, for the unilateral poaching equilibrium of platform B, the marginal consumer is still located at $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$.

From the profits of two platforms in the second period and the marginal consumer $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$, the two-period discounted profits Π_A and Π_B of two platforms expressed in terms of the prices in the first period are shown as follows

$$\Pi_A=\begin{cases}\frac{p_{1A}(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{9t(1-\lambda)}{16(1-\alpha\lambda)}, & \text{if } p_{1B}-\frac{t}{1-\alpha\lambda}\leq p_{1A}<p_{1B}-\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{1-\alpha\lambda}\\ \frac{(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]\{\delta_f[4(v+\alpha\beta)-t-(1-\alpha\lambda)p_{1B}]+(4+\delta_f)(1-\alpha\lambda)p_{1A}\}}{8t(1-\alpha\lambda)}, & \text{if } p_{1B}-\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)}\leq p_{1A}\leq p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}\\ \frac{p_{1A}(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{t(1-\lambda)}{8(1-\alpha\lambda)}, & \text{if } p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}<p_{1A}\leq p_{1B}+\frac{t}{(1-\alpha\lambda)}\end{cases} \tag{10}$$

$$
\Pi_B=\begin{cases}
\frac{(1-\lambda)p_{1B}[t-(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{9t(1-\lambda)}{16(1-\alpha\lambda)}, \text{ if } \\
p_{1A}-\frac{t}{1-\alpha\lambda}\le p_{1B}<p_{1A}-\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{1-\alpha\lambda} \\
\frac{(1-\lambda)[t+(1-\alpha\lambda)(p_{1A}-p_{1B})]\{\delta_f[4(v+\alpha\beta)-t-(1-\alpha\lambda)p_{1A}]+(4+\delta_f)(1-\alpha\lambda)p_{1B}\}}{8t(1-\alpha\lambda)}, \text{ if} \\
p_{1A}-\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}\le p_{1B}\le p_{1A}+\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)} \\
\frac{(1-\lambda)p_{1B}[t-(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{t(1-\lambda)}{8(1-\alpha\lambda)}, \text{ if } \\
p_{1A}+\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{1-\alpha\lambda}<p_{1B}\le p_{1A}+\frac{t}{1-\alpha\lambda}
\end{cases}
$$

Next, we derive the equilibrium prices. When two platforms make decisions in the first period, they maximize profits by choosing the prices p_{1A} and p_{1B} in the first period. Therefore, it is necessary to solve the price reaction functions of two platforms under each equilibrium in the second period, and then we obtain the equilibrium prices.

For the unilateral poaching equilibrium of platform A, platform A maximizes its profits by $\Pi_A=\frac{p_{1A}(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{t(1-\lambda)}{8(1-\alpha\lambda)}$ with the constraint $p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}<p_{1A}\le p_{1B}+\frac{t}{(1-\alpha\lambda)}$. Platform B maximizes its profits by $\Pi_B=\frac{(1-\lambda)p_{1B}[t-(1-\alpha\lambda)(p_{1B}-p_{1A})]}{2t}+\delta_f\frac{9t(1-\lambda)}{16(1-\alpha\lambda)}$ with the constraint $p_{1A}-\frac{t}{1-\alpha\lambda}\le p_{1B}<p_{1A}-\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{1-\alpha\lambda}$. By the Kuhn-Tucker conditions, the price-reaction functions of two platforms are as follows

$$
p_{1A}(p_{1B})=\begin{cases}
\frac{t+p_{1B}(1-\alpha\lambda)}{2(1-\alpha\lambda)}, \text{if} -\frac{t}{1-\alpha\lambda}\le p_{1B}<\frac{t-2\left[t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}\right]}{1-\alpha\lambda} \\
p_{1B}+\frac{t}{(1-\alpha\lambda)}, \quad \text{if } p_{1B}\le-\frac{t}{1-\alpha\lambda}
\end{cases}
$$

$$
p_{1B}(p_{1A})=\begin{cases}
\frac{t+p_{1A}(1-\alpha\lambda)}{2(1-\alpha\lambda)}, \text{ if } \frac{t+2\left[t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}\right]}{1-\alpha\lambda}<p_{1A}\le\frac{3t}{(1-\alpha\lambda)} \\
p_{1A}-\frac{t}{1-\alpha\lambda}, \text{ if } p_{1A}\ge\frac{3t}{1-\alpha\lambda}
\end{cases}
$$

From the reaction functions of the price for two platforms shows that the equilibrium lies outside the constraint range. Therefore, there exists no equilibrium that can maximize the profits of both platforms simultaneously in this case.

For the PPD equilibrium, we derive that platform A maximizes its profits by $\Pi_A=\frac{(1-\lambda)[t+(1-\alpha\lambda)(p_{1B}-p_{1A})]\{\delta_f[4(v+\alpha\beta)-t-(1-\alpha\lambda)p_{1B}]+(4+\delta_f)(1-\alpha\lambda)p_{1A}\}}{8t(1-\alpha\lambda)}$ with the constraint $p_{1B}-\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)}\le p_{1A}\le p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}$. platform B maximizes its profits by $\Pi_B=\frac{(1-\lambda)[t+(1-\alpha\lambda)(p_{1A}-p_{1B})]\{\delta_f[4(v+\alpha\beta)-t-(1-\alpha\lambda)p_{1A}]+(4+\delta_f)(1-\alpha\lambda)p_{1B}\}}{8t(1-\alpha\lambda)}$ with the constraint $p_{1A}-\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}\le p_{1B}\le p_{1A}+\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)}$. By the Kuhn-Tucker conditions, the reaction function of the price for two platforms are respectively

$$
p_{1A}(p_{1B})=\begin{cases}
p_{1B}+\frac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}, \quad \text{if } p_{1B}\le\frac{8(v+\alpha\beta)-2t-(4+\delta_f)\left[\sqrt{4(v+\alpha\beta)^2-t^2}\right]}{2(1-\alpha\lambda)} \\
\frac{2t+(2+\delta_f)(1-\alpha\lambda)p_{1B}+\delta_f[t-2(v+\alpha\beta)]}{(1-\alpha\lambda)(4+\delta_f)}, \text{ if} \\
\frac{8(v+\alpha\beta)-2t-(4+\delta_f)\sqrt{4(v+\alpha\beta)^2-t^2}}{2(1-\alpha\lambda)}<p_{1B}<\frac{2t(3+\delta_f)-4(v+\alpha\beta)(2+\delta_f)+(4+\delta_f)\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2(1-\alpha\lambda)} \\
p_{1B}-\frac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)}, \quad \text{if } p_{1B}\ge\frac{2t(3+\delta_f)-4(v+\alpha\beta)(2+\delta_f)+(4+\delta_f)\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2(1-\alpha\lambda)}
\end{cases} \quad (14)
$$

(15)

(16)

$$p_{1B}(p_{1A})=\begin{cases} p_{1A}+\dfrac{t-2(v+\alpha\beta)+\sqrt{[4(v+\alpha\beta)^2-t^2]}}{(1-\alpha\lambda)} & ,\text{if } p_{1A}\leq\dfrac{8(v+\alpha\beta)-2t-(4+\delta_f)\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2(1-\alpha\lambda)} \\ \dfrac{2t+(2+\delta_f)(1-\alpha\lambda)p_{1A}+\delta_f[t-2(v+\alpha\beta)]}{(1-\alpha\lambda)(4+\delta_f)} & ,\text{ if } \\ \dfrac{8(v+\alpha\beta)-2t-(4+\delta_f)\sqrt{[4(v+\alpha\beta)^2-t^2]}}{2(1-\alpha\lambda)}<p_{1A}<\dfrac{2t(3+\delta_f)-4(v+\alpha\beta)(2+\delta_f)+(4+\delta_f)\sqrt{4(v+\alpha\beta)^2-t^2}}{2(1-\alpha\lambda)} & \\ p_{1A}-\dfrac{t-2(v+\alpha\beta)+\sqrt{4(v+\alpha\beta)^2-t^2}}{(1-\alpha\lambda)}, & \text{if } p_{1A}\geq\dfrac{2t(3+\delta_f)-4(v+\alpha\beta)(2+\delta_f)+(4+\delta_f)\sqrt{4(v+\alpha\beta)^2-t^2}}{2(1-\alpha\lambda)} \end{cases}$$

By simultaneously solving the reaction functions of price for two platforms, the equilibrium prices in the first period are given by

$$p_{1A}^{a}=p_{1B}^{a}=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$$

Similarly, we derive the poaching equilibrium of platform B. The equilibrium still lies outside the constraint range, and there is still no equilibrium that can maximize the profits of both platforms simultaneously.

Finally, we obtain the equilibrium prices of two platforms in the first period are $p_{1A}^{a}=p_{1B}^{a}=\frac{2t+\delta_f[t-2(v+\alpha\beta)]}{2(1-\alpha\lambda)}$. By Substituting it into the marginal consumer formula $z=\frac{1}{2}+\frac{(1-\alpha\lambda)(p_{1B}-p_{1A})}{2t}$, we obtain $z^{a}=\frac{1}{2}$. In the second period, two platforms reach the PPD equilibrium. They both set sufficiently high uniform prices $p_{2AU},p_{2BU}\geq\frac{v+\alpha\beta}{1-\alpha\lambda}$. Platform A serves consumers in the interval $\left[0,\frac{1}{2}\right]$ with the personalized price $p_{2A}^{a}(x)=\frac{v-tx+\alpha\beta}{1-\alpha\lambda}$, and platform B serves consumers in the interval $\left[\frac{1}{2},1\right]$ with the personalized price $p_{2B}^{a}(y)=\frac{v-t(1-y)+\alpha\beta}{1-\alpha\lambda}$.

Appendix 3: Proof of Proposition 7

When there exists a cost ($0<c\leq v$) for the consumer identity management, we analyze whether two platforms will deviate from the PPD equilibrium in the second period.

Case 1: $0\leq z\leq\frac{1}{2}$

As can be seen from the previous analysis, due to the non-competitive interval, platform B has no incentive to deviate from the PPD equilibrium at this time. The only reason for platform A to deviate from the PPD equilibrium is to poach some consumers of platform B in the interval $\left[z,\frac{1}{2}\right]$. Suppose platform A deviates from the PPD equilibrium by decreasing the uniform price to $p_{2AU}<\frac{v+\alpha\beta}{1-\alpha\lambda}-c$. This will make some of platform A's old consumers to manage their identities and become active consumers.

Let the marginal old consumer $\hat{x}\in[0,z]$ be indifferent between paying the PPD price of platform A and the uniform price $p_{2AU}+c$, that is $v-t\hat{x}-p_{2A}(\hat{x})+\alpha(\beta+\lambda p_{2A}(\hat{x}))=v-t\hat{x}-(p_{2AU}+c)+\alpha[\beta+\lambda(p_{2AU}+c)]$. By $p_{2A}(\hat{x})=\frac{v-t\hat{x}+\alpha\beta}{1-\alpha\lambda}$ we have $\hat{x}=\frac{v+\alpha\beta-(p_{2AU}+c)(1-\alpha\lambda)}{t}$. Therefore, platform A set the personalized price $p_{2A}(x)=\min\{p_{2A}^{PPD},p_{2AU}+c\}$ for all its old consumers. At this time, consumers in $[0,\hat{x}]$ pay $p_{2AU}+c$, and consumers in $[\hat{x},z]$ pay the PPD price.

Let the marginal consumer $\tilde{y}\in\left[z,\frac{1}{2}\right]$ be indifferent between purchasing from platform A and from platform B. Given p_{2AU}, the minimum price set by platform B for consumers $p_{2B}(y)=0$. Therefore, for the marginal consumer $\tilde{y}$, we have $v-t\tilde{y}-p_{2AU}+\alpha(\beta+\lambda p_{2AU})=v-t(1-\tilde{y})-0+\alpha(\beta+0)$, and then $\tilde{y}=\frac{1}{2}-\frac{p_{2AU}(1-\alpha\lambda)}{2t}$.

The relationship between the marginal consumers $\hat{x}$ and z is $\hat{x}\leq z$. The deviation profit of Platform A is $\pi_{2A}^{d}=\int_0^{\min\{\hat{x},z\}}(1-\lambda)(p_{2AU}+c)dx+\int_{\min\{\hat{x},z\}}^{z}(1-\lambda)\left(\frac{v-tx+\alpha\beta}{1-\alpha\lambda}\right)dx+(1-\lambda)p_{2AU}(\tilde{y}-z)$. Therefore, it is necessary to discuss under two scenarios where $\hat{x}<z$ and $\hat{x}=z$.

(i) Some of the old consumers choose to manage their identities, i.e., $\hat{x}<z$.

The situation at this time can be represented by Figure A.1.

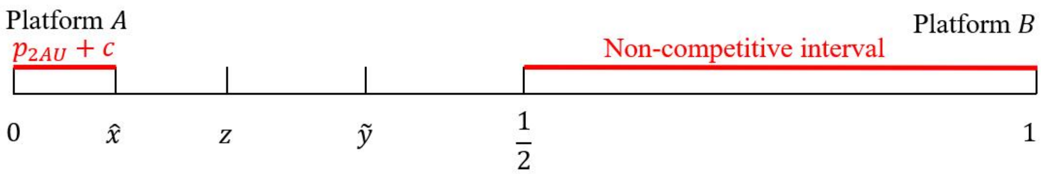

Figure A.1 Some Old Consumers Choose the Identity Management when $0\leq z\leq\frac{1}{2}$

The deviation profit of platform A is $\pi_{2A}^{d}=\int_0^{\hat{x}}(1-\lambda)(p_{2AU}+c)dx+\int_{\hat{x}}^{z}(1-\lambda)\left(\frac{v-tx+\alpha\beta}{1-\alpha\lambda}\right)dx+(1-\lambda)p_{2AU}(\tilde{y}-z)$. Substitute

$\hat{x}=\frac{v+\alpha\beta-(p_{2AU}+c)(1-\alpha\lambda)}{t}$ and $\tilde{y}=\frac{1}{2}-\frac{p_{2AU}(1-\alpha\lambda)}{2t}$ into the equation, then we obtain the optimal deviation price of platform A $p^d_{2AU}=\frac{2(v+\alpha\beta)+t(1-2z)-2c(1-\alpha\lambda)}{4(1-\alpha\lambda)}$, where $\hat{x}=\frac{2(v+\alpha\beta)-t(1-2z)-2c(1-\alpha\lambda)}{4t}$, $\tilde{y}=\frac{t(3+2z)-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{8t}$. Since $\hat{x}<z$, we have $\frac{2(v+\alpha\beta)-t-2c(1-\alpha\lambda)}{2t}<z$. Also, from $z\leq\tilde{y}$, we obtain $z\leq\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{6t}$. By $\frac{2(v+\alpha\beta)-t-2c(1-\alpha\lambda)}{2t}<$ and $z\leq\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{6t}$, we have $\frac{2(v+\alpha\beta)-t-2c(1-\alpha\lambda)}{2t}<\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{6t}$. Then we have $\frac{4(v+\alpha\beta)-3t}{4(1-\alpha\lambda)}<c\leq v$. The optimal deviation profit is $\pi^d_{2A}=\frac{(1-\lambda)}{16t(1-\alpha\lambda)}\{t^2(1-4z-4z^2)+4t(1+2z)(v+\alpha\beta)-4(v+\alpha\beta)^2+4c(1-\alpha\lambda)[t(2z-1)+2(v+\alpha\beta)]-4c^2(-1+\alpha\lambda)^2\}$. The PPD profit of platform A is $\pi^{PPD}_{2A}=\int_0^z\left[\frac{v-tx+\alpha\beta}{1-\alpha\lambda}(1-\lambda)\right]dx=\frac{(1-\lambda)}{1-\alpha\lambda}\left[(v+\alpha\beta)z-\frac{1}{2}tz^2\right]$.

By comparing the two profits and the condition $0\leq z\leq\frac{1}{2}$, we have that platform A will not deviate from the PPD equilibrium when $\frac{1}{2}+\frac{[v+\alpha\beta-c(1-\alpha\lambda)](1-\sqrt{2})}{t}\leq z\leq\frac{1}{2}$.

(ii) All old consumers choose the identity management, i.e., $\hat{x}=z$.

At this time, $\pi^d_{2A}=\int_0^z(1-\lambda)(p_{2AU}+c)dx+(1-\lambda)p_{2AU}(\tilde{y}-z)$. By optimizing the profits under constraints, we obtain $p^d_{2AU}=\frac{t}{2(1-\alpha\lambda)}$, $\hat{x}=\frac{v+\alpha\beta-\left(\frac{t}{2(1-\alpha\lambda)}+c\right)(1-\alpha\lambda)}{t}=z\leq\tilde{y}=\frac{1}{2}-\frac{\frac{t}{2(1-\alpha\lambda)}(1-\alpha\lambda)}{2t}$. Thus, we derive $\frac{4(v+\alpha\beta)-3t}{4(1-\alpha\lambda)}\leq c\leq v$. The equilibrium deviation profit is $\pi^d_{2A}=(1-\lambda)\left[\frac{t}{8(1-\alpha\lambda)}+cz\right]$.

By comparing the deviation profit and the PPD profit, we can find that when $\frac{2[v+\alpha\beta-c(1-\alpha\lambda)]-\sqrt{4[v+\alpha\beta-c(1-\alpha\lambda)]^2-t^2}}{2t}\leq z\leq\frac{1}{2}$, platform A will not deviate from the PPD equilibrium.

Case 2: $\frac{1}{2}\leq z\leq 1$

At this time, platform A will not deviate from the PPD equilibrium, so we need to analyze whether platform B will deviate from the PPD equilibrium. Let the marginal consumer $\hat{y}\in[z,1]$ be indifferent between paying the PPD price and the uniform price $p_{2BU}+c$, that is $v-t(1-\hat{y})-p_{2B}(\hat{y})+\alpha\big(\beta+\lambda p_{2B}(\hat{y})\big)=v-t(1-\hat{y})-(p_{2BU}+c)+\alpha[\beta+\lambda(p_{2BU}+c)]$. Then we have $\hat{y}=\frac{(p_{2BU}+c)(1-\alpha\lambda)+t-(v+\alpha\beta)}{t}$. Let the marginal consumer $\tilde{x}\in\left[\frac{1}{2},z\right]$ be indifferent between purchasing from platform A and from platform B, that is $v-t\tilde{x}-0+\alpha(\beta+0)=v-t(1-\tilde{x})-p_{2BU}+\alpha(\beta+\lambda p_{2BU})$. Then we have $\tilde{x}=\frac{1}{2}+\frac{p_{2BU}(1-\alpha\lambda)}{2t}$.

Based on the analysis of the Case 1, the results are obtained as follows

(1) Some of the old consumers choose to manage their identities, i.e., $z<\hat{y}$.

The situation at this time can be represented by Figure A.2.

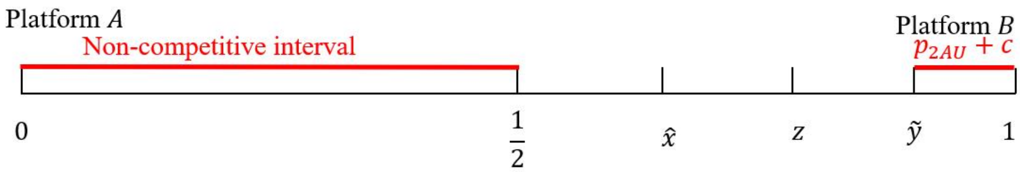

Figure A.2 Some Old Consumers Choose the Identity Management when $\frac{1}{2}\leq z\leq 1$

Then we have $p_{2BU}=\frac{2[v+\alpha\beta-c(1-\alpha\lambda)]+t(2z-1)}{4(1-\alpha\lambda)}$, $\hat{y}=\frac{t(3+2z)-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{4t}$ and $\tilde{x}=\frac{t(3+2z)+2(v+\alpha\beta)-2c(1-\alpha\lambda)}{8t}$. Since $z<\hat{y}$, we derive $z<\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{2t}$. Given $\tilde{x}\leq z$, we have $\frac{3t+2(v+\alpha\beta)-2c(1-\alpha\lambda)}{6t}\leq z$. By $z<\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{2t}$ and $\frac{3t+2(v+\alpha\beta)-2c(1-\alpha\lambda)}{6t}\leq z$, we have $\frac{3t+2(v+\alpha\beta)-2c(1-\alpha\lambda)}{6t}<\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{2t}$, i.e., $\frac{4(v+\alpha\beta)-3t}{4(1-\alpha\lambda)}<c\leq v$.

The equilibrium deviation profit of platform B is $\pi^d_{2B}=\frac{(1-\lambda)}{16t(1-\alpha\lambda)}\{t^2(-7+12z-4z^2)+4t(3-2z)(v+\alpha\beta)-4(v+\alpha\beta)^2+4c(1-\alpha\lambda)[2(v+\alpha\beta)+t(1-2z)]-4c^2(-1+\alpha\lambda)^2\}$. The PPD profit is $\pi^{ppd}_{2B}=\frac{(1-\lambda)}{1-\alpha\lambda}\left[v+\alpha\beta-\frac{1}{2}t-z(v+\alpha\beta-t)-\frac{1}{2}tz^2\right]$. By comparing the deviation profits and the PPD profits, we have that Platform B will not deviate from the PPD equilibrium when $\frac{1}{2}\leq z\leq\frac{1}{2}-\frac{[v+\alpha\beta-c(1-\alpha\lambda)](1-\sqrt{2})}{t}$.

(ii) All old consumers choose the identity management, i.e., $z=\hat{y}$.

At this time, we have $p_{2BU}=\frac{t}{2(1-\alpha\lambda)}$, $\hat{y}=\frac{3t-2(v+\alpha\beta)+2c(1-\alpha\lambda)}{2t}$ and $\tilde{x}=\frac{3}{4}$. Since $z=\hat{y}\geq\frac{3}{4}$, we have $\frac{4(v+\alpha\beta)-3t}{4(1-\alpha\lambda)}\leq c\leq v$. And the equilibrium deviation profit is $\pi^d_{2B}=\frac{(1-\lambda)[t+8c(1-\alpha\lambda)(1-z)]}{8(1-\alpha\lambda)}$. By comparing the deviation profit and the PPD profit, we can find that when $\frac{1}{2}\leq z\leq\frac{-2(v+\alpha\beta-t-c(1-\alpha\lambda))+\sqrt{4[v+\alpha\beta-c(1-\alpha\lambda)]^2-t^2}}{2t}$, platform A will not deviate from the PPD equilibrium.

Finally, By Case 1 and 2 we have that the PPD equilibrium exists if and only if $z\in\left[\frac{1}{2}+\frac{[v+\alpha\beta-c(1-\alpha\lambda)](1-\sqrt{2})}{t},\frac{1}{2}-\frac{[v+\alpha\beta-c(1-\alpha\lambda)](1-\sqrt{2})}{t}\right]$ when some old consumers conduct the identity management. On the contrary, when all consumers conduct the identity management, the PPD equilibrium exists if and only if

$z \in \left[\frac{2[v+\alpha\beta-c(1-\alpha\lambda)]-\sqrt{4[v+\alpha\beta-c(1-\alpha\lambda)]^2-t^2}}{2t}, \frac{-2(v+\alpha\beta-t-c(1-\alpha\lambda))+\sqrt{4[v+\alpha\beta-c(1-\alpha\lambda)]^2-t^2}}{2t}\right]$. The interval decreases of z in c, which indicates that the smaller of c, the more likely the PPD equilibrium will be achieved.