World Journal of Engineering Research
Print ISSN: 2959-9865
Online ISSN: 2959-9873
DOI: https://doi.org/10.61784/wjer3086

A STATISTICAL MICROMECHANICS FRAMEWORK FOR PIEZORESISTIVITY IN VISCOELASTIC FOAMS: UNIFYING STRUCTURAL EVOLUTION FROM CRACK TO CONTACT REGIMES

Jie Lv
Shaoxing Universtiy, Shaoxing 312000, Zhejiang, China.

Abstract: We introduce a comprehensive computational framework to simulate the complex electro-mechanical response of flexible, porous piezoresistive sensors. Existing simulation approaches face a critical bottleneck: they either rely on homogenized material properties that ignore the determinative porous micro-architecture, or they employ simplified, idealized geometries that fail to capture the stochastic nature of real foams. Most critically, prior models have overwhelmingly neglected the intrinsic viscoelasticity of the polymer matrix, a fundamental property governing the material's time-dependent mechanical response. Our framework overcomes these limitations by integrating three key elements within a Finite Element (FE) environment: (1) A stochastic 2D Representative Volume Element (RVE) based on a novel, computationally-stable abstracted pore geometry; (2) A Generalized Maxwell model to capture the experimentally-verified viscoelastic stress relaxation of the polyurethane (PU) substrate; and (3) A "segmented quasi-static" (SQS) solution strategy, developed to circumvent the documented limitations of commercial FE software in handling fully-coupled dynamic, conductive-contact simulations. The model is built by generating an ensemble of over 200 RVEs, each with randomly-distributed internal pore geometries, to represent the foam's statistical heterogeneity. By performing an ensemble average of the simulation results, our model successfully generates a macroscopic resistance-strain curve that is in high agreement with experimental observations. Crucially, this work provides a unified, structure-based origin for the two distinct sensing regimes widely reported in the literature: the initial, high-sensitivity "crack effect" (GF > 25) is shown to be a statistical emergent property of the initial contact events of the most vulnerable pores in the stochastic distribution, while the stable, high-strain "contact effect" corresponds to the progressive, large-scale pore collapse and contact area growth across the ensemble. This framework bridges the gap between stochastic microstructure and predictable macroscopic performance, opening a pathway for the inverse design of foam-based sensors.

Keywords: COMSOL; Representative Volume Element; Segmented quasi-static

1 INTRODUCTION

Flexible, compressible piezoresistive sensors are a cornerstone technology for next-generation applications, including electronic skin (e-skin), wearable health monitoring, and advanced human-machine interfaces. Unlike traditional rigid silicon-based sensors, which are brittle and ill-suited for bodily contact, sensors fabricated from conductive polymer foams (e.g., polyurethane (PU) sponges coated with graphene, CNTs, or AgNWs) offer unparalleled flexibility, high compressibility, and exceptional sensitivity to the low-pressure regimes relevant to human touch (e.g., 0.5–100 kPa).

The sensing mechanism in these materials is fundamentally structural. As the foam is compressed, its complex porous architecture deforms, causing internal conductive pathways to buckle, bend, and reconfigure. New conductive contacts are formed as pore walls touch, while existing pathways may be altered or broken. The macroscopic change in resistance is therefore a direct consequence of the evolution of this internal conductive network [1].

1.1 The Experimental Target: A Two-Regime Piezoresistive Response

A large body of experimental work has established a characteristic, two-regime response for these porous sensors, which any high-fidelity model must successfully replicate. This behavior is most clearly defined by the Gauge Factor (GF), GF = $(\Delta R/R\_0)/\varepsilon$.

The "Crack Effect" (Low-Strain, High-Sensitivity): In the initial, very low strain regime (e.g., $\varepsilon < 0.6\%$), sensors exhibit an "ultra-high" sensitivity, with reported GFs as high as 26 [2]. This phenomenon is often attributed to the rapid opening or closing of pre-existing micro-cracks in the conductive coating [3]. The "fractured microstructure design" proposed by Liu et al. is a premier example of leveraging this effect to achieve extreme sensitivity (0.26 kPa^{-1}) [4].

The "Contact Effect" (High-Strain, Stable-Sensitivity): At larger strains, the GF typically decreases significantly but stabilizes, providing a wide, reliable operating range for the sensor [2]. This regime is understood to be dominated by the progressive collapse of pores and the corresponding increase in the total contact area between adjacent conductive surfaces [5].

A persistent challenge in the field has been the lack of a unified computational model that can capture both regimes and, critically, explain the transition between them as a continuous evolution of a single underlying microstructure, rather than as two decoupled physical mechanisms.

1.2 The Critical Gap in Computational Modeling

While the experimental understanding is clear, computational modeling has lagged, primarily due to a reliance on incomplete or inappropriate physical assumptions. As outlined in our preliminary work, the field suffers from three principal flaws:

Critique 1: Homogenized Material-Property Models. Many simulations, such as the master's thesis by Yan, treat the conductive composite as a homogenized medium [6]. These models, often based on Monte Carlo percolation or tunneling theories, are effective at describing tunneling phenomena but fundamentally ignore the explicit, complex pore-structure mechanics (i.e., how the foam itself deforms and collapses) that govern the sensor's response [7].

Critique 2: Oversimplified Geometric Models. Other attempts using the Finite Element Method (FEM), such as the work by Shen et al., employ overly-simplified unit-cell geometries, such as "simple cubic structures"[2]. These idealized, perfectly periodic models (e.g., Kelvin or cubic cells) bear little resemblance to the stochastic, disordered topology of real foams and thus cannot capture the statistical nature of pore collapse [8].

Critique 3: The Missing Physics—Viscoelasticity. Perhaps the most significant omission in prior work is the failure to account for the viscoelasticity of the polymer substrate. PU is a polymer, and at room temperature, its mechanical response is dominated by time-dependent behaviors such as stress relaxation and creep. As our experimental data confirms, any model treating the foam as purely hyper-elastic is fundamentally incorrect and cannot capture the sensor's dynamic or long-term behavior. While some advanced continuum models have begun to incorporate viscoelasticity (e.g., using a Generalized Maxwell model within beam theory), they still lack the 2D/3D micro-structural fidelity [5].

1.3 Our Framework: A Statistical, Viscoelastic, Structural Model

This paper introduces a computational framework designed to simultaneously address all three of these deficiencies. We present a structure-based, viscoelastic, and statistical micromechanics model that captures the true physics of the piezoresistive foam. This paper details:

The development of a 2D stochastic Representative Volume Element (RVE) that is physically abstracted for computational convergence but statistically representative of foam topology.

The full integration of a Generalized Maxwell viscoelastic constitutive model, with parameters fit directly from experimental stress-relaxation data.

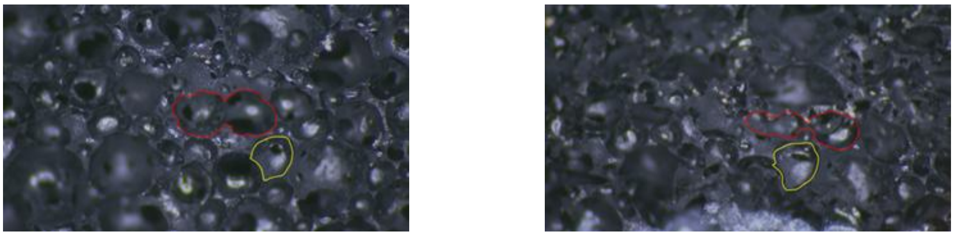
A novel "segmented quasi-static" (SQS) solver strategy, designed to overcome the limitations of standard FEM software in coupled dynamic-contact-conduction problems.

We will demonstrate that by performing an ensemble average over 200+ stochastic RVE simulations, our model generates a macroscopic sensor response curve that not only matches experimental results but, more importantly, provides a unified, micro-structural explanation for the "crack" and "contact" effects.

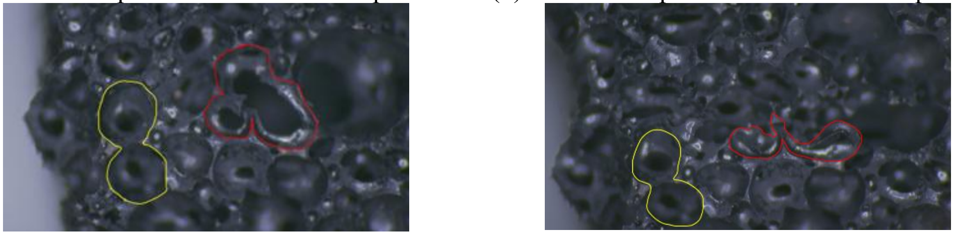
2 MODEL DEVELOPMENT I: THE STOCHASTIC RVE

2.1 From Physical Observation to Geometric Abstraction

The model's foundation must be the physical micro-architecture. Analysis of the (implied) SEM imagery of compressed PU foam reveals the "weakest" structural elements that deform first. This critical unit was identified as an "∞"-shaped (infinity) pore structure. During compression, this elongated, centrally-constricted pore is the first to buckle, bend, and collapse, forming a new contact interface at its center.(Figure 1)



(a) Before Compression and After Compression (b) Before Compression and After Compression



(c) Before Compression and After Compression (d) Before Compression and After Compression

Figure 1 Comparison of Pore Compression Before and After Squeezing of Sponge

A preliminary model based on this "∞" shape was built and confirmed to be superior to a simpler "8"-shape, as the "∞" model effectively increases contact area upon compression while the "8" model does not.

2.2 The Computational Bottleneck: Singularity and Instability

Despite being physically representative, the "∞" geometry proved to be computationally intractable for a coupled electro-mechanical simulation. This failure stemmed from two distinct numerical issues:

Electrical Singularity: The center of the "∞" model forms a "tip" or a sharp point. When this tip makes contact with the opposing surface, the contact area is mathematically zero. This creates a singularity in the electrical solver, preventing convergence. As noted in the development log, this "could not provide a basis for subsequent electrical module calculations".

Mechanical Instability: The "circular frame" of the "∞" model was found to be mechanically unstable under large deformation. The simulation "results could not converge," likely due to severe mesh distortion or a non-physical lateral shift of the unconstrained curved surfaces.

This trade-off between physical fidelity and numerical stability is a well-known challenge in finite element modeling.

2.3 The Solution: A Computationally-Robust, Abstracted RVE

To resolve this, we developed a novel, abstracted RVE geometry that preserves the essential physics (pore collapse and contact area growth) while ensuring numerical robustness. (Figure 2)

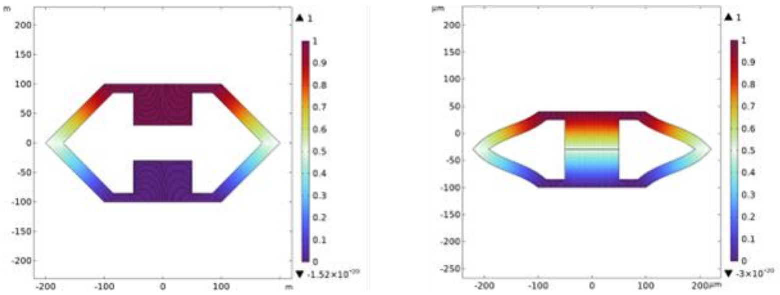


Figure 2 Model Shape

Abstraction 1: "Circular Frame"→"Hexagonal Frame"

This change was made to "ensure the model does not shift left or right after compression". A hexagonal boundary is a standard technique in RVE modeling 15 that allows for proper application of boundary conditions (e.g., symmetry or periodic) to simulate uniaxial compression without rigid body motion.

Abstraction 2: "Tip" →"Rectangular Protrusion"

This is the most critical abstraction. The sharp tip was replaced with a flat-topped rectangle to "ensure sufficient area after contact for the electric potential to drop across the contact surface". This regularizes the contact problem, transforming it from a mathematically ill-posed "point-to-surface" contact into a numerically stable and solvable "surface-to-surface" contact.

2.4 Introducing Stochasticity: From a Single Pore to a Foam

A single RVE, no matter how well-designed, does not represent a foam. A foam is a stochastic material whose properties arise from the collective behavior of thousands of different pores [9].

Implementation: We used MATLAB to generate a "random hexagonal grid" representing the RVE ensemble. (Figure 3)

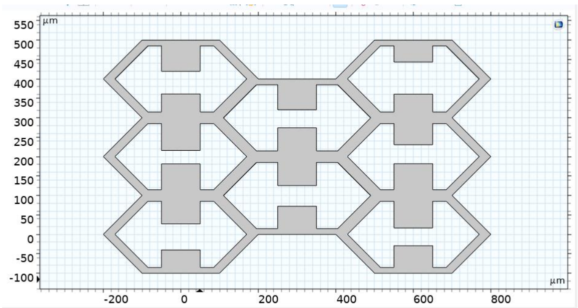


Figure 3 Random Hexagonal Grid

The Key Mechanism: Within each hexagonal RVE, the "length of the protrusion is randomly generated".
Physical Significance: This distribution of protrusion lengths (and thus, internal gap sizes) is the central mechanism by which our model simulates the foam's piezoresistive response. At any given compressive strain ε, only a subset of the RVEs in the ensemble (those with the smallest gaps) will have established electrical contact. This statistical ensemble is the link between the micro-geometry and the macro-response.
Model Scale: The final analysis is based on an ensemble average of "200+" models.
Choice of 2D: The decision to use 2D RVEs instead of 3D was a strategic computational trade-off "to obtain more samples" in a limited time, operating on the hypothesis that the statistical average of a large 2D ensemble can effectively approximate the behavior of a 3D stochastic foam.

3 MODEL DEVELOPMENT II: MULTIPHYSICS IMPLEMENTATION

3.1 Mechanical Model: Finite-Strain Viscoelasticity

As established in Critique 3, treating the PU substrate as a simple elastic solid is physically incorrect.
Experimental Justification: Our own experimental mechanical testing clearly demonstrates stress relaxation. When the foam is held at a constant compressive strain, the measured reaction force decays over time. This is the hallmark of a viscoelastic material.
Constitutive Model Selection: A simple Maxwell model (one spring, one dashpot) is insufficient, as it "has only one relaxation time" and cannot capture the complex relaxation spectrum of a polymer.
Therefore, we selected the Generalized Maxwell Model (or Wiechert model), which consists of multiple Maxwell elements in parallel with a parallel equilibrium spring. This is a standard and robust choice for modeling polymer mechanics.5
Governing Equation: The time-dependent relaxation modulus E(t) is given by: $E(t) = E_\infty+\sum E_i e^{(-t/\tau_i)}$
where E_∞ is the equilibrium modulus (the stiffness as $t \to \infty$), and Ei and τ_i are the stiffness and relaxation time of the i-th Maxwell element, respectively.
Parameterization: This equation was fit to our experimental stress-relaxation curve to extract the set of parameters ($E_\infty+\sum E_i e^{(-t/\tau_i)}$ {i=1...n}) that define our specific PU material. These parameters, listed in Table 1, form the empirical basis of the mechanical simulation. (Figure 4)

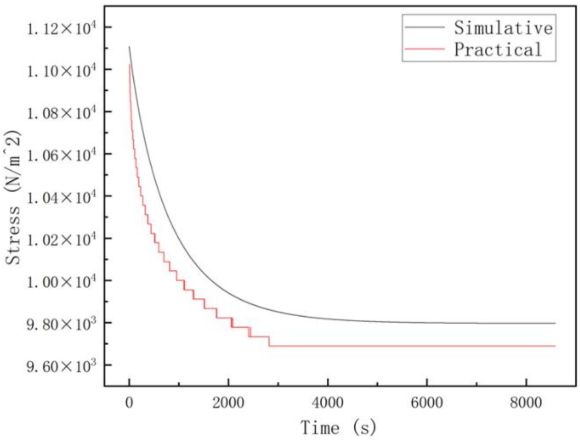


Figure 4 Stress Relaxation Comparison Curve

FEM Implementation: This constitutive model was implemented within the COMSOL Multiphysics Solid Mechanics module, coupled with a finite-strain formulation to accurately capture the large deformations and time-dependent pore collapse of the RVE geometry.

Table 1 Fitted Viscoelastic Parameters for Generalized Maxwell Model. (Derived from experimental stress-relaxation data)

Maxwell Element (i)	Modulus Ei (Pa)	Relaxation Time τi (s)
1	5.25e8	8
2	5.95e8	15
3	6.65e8	25
4	4.2e8	900
5	4.2e8	10000

3.2 Electro-Mechanical Coupling: The "Segmented Quasi-Static" (SQS) Strategy

The final objective is a fully coupled electro-mechanical simulation. However, this presents a significant challenge.

The Specific Limitation: As identified in our primary source, "COMSOL currently does not support dynamic real-time calculation for the contact conductive mode". This limitation is common in commercial FEM software, as solving for dynamic mechanical deformation (e.g., viscoelasticity) simultaneously with a changing-contact boundary condition (where the electrical pathway is created or destroyed) is numerically unstable.

Our Innovative Workaround: To bypass this limitation, we developed and implemented a "segmented static calculation" or "segmented quasi-static" (SQS) approach. This strategy effectively decouples the mechanical and electrical problems in the time domain.

The SQS workflow is as follows:

Step 1: Dynamic Mechanical Simulation. First, a purely mechanical, transient (dynamic) simulation is run. This simulation uses the full Generalized Maxwell model (Table 1) to solve for the RVE's deformation, and pore collapse over the full time period (e.g., t=0 to t=1.0s).

Step 2: Export Deformed Geometries. From this single dynamic simulation, we export the deformed geometric state (i.e., the displaced mesh) at several discrete time steps (t_1, t_2,..., t_n).

Step 3: Segmented Static Electrical Simulations. Each of these t_n exported, deformed, and static geometries is then used as the input for a separate, steady-state (static) electrical simulation (e.g., COMSOL "Electric Currents" physics). A voltage (e.g., 1V) is applied between the top and bottom boundaries, and the total current I is calculated for that specific deformed state.

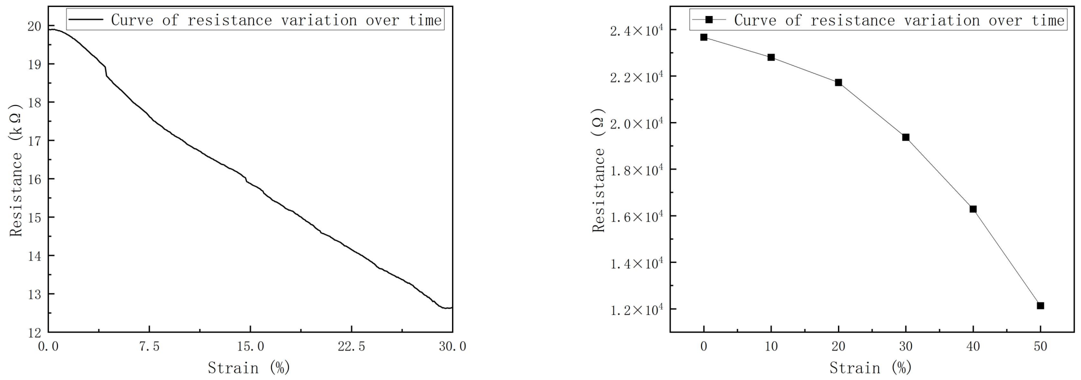
Step 4: Curve Reconstruction. The resistance for each time step is calculated via Ohm's Law, R(t_i) = V/I(t_i). These discrete resistance points are then plotted against their corresponding strain ε (t_i) to reconstruct the full, time-dependent piezoresistive response curve.

This SQS method is a powerful and novel workaround, enabling the solution of a complex, dynamic, coupled-field problem using only the standard static solvers available in commercial FEM packages.

4 RESULTS AND DISCUSSION

4.1 Statistical Variance vs. Homogenized Response

The first key result is seen in the raw output from the 200+ individual RVE simulations, as schematically represented in the "Resistance vs. Deformation Curve" plot (figure 5). This plot shows a high degree of variance; different, randomly-generated RVEs produce widely different resistance-strain curves.



(a) Actual Resistance Curve (b) Simulated Resistance Curve

Figure 5 The Curve of Resistance Variation over Time

This variance is not computational noise—it is the physical reality of a stochastic material. At the microscopic level, the foam's response is heterogeneous and unpredictable. This observation confirms why models based on a single, idealized geometry (as in Critique 2) are guaranteed to fail: they capture only one arbitrary instance from a wide statistical distribution.

The final predictive result of our framework is the "Average Resistance vs. Deformation Curve". This curve is the ensemble average of the 200+ individual simulations. This smooth, predictable curve represents the emergent, macroscopic, and homogenized sensor response [10]. It is the central validation of our statistical RVE approach 9, demonstrating how a predictable macro-property emerges from the average of many random micro-scale events.

4.2 A Unified Micro-Structural Origin for the Two-Regime Response

The averaged macro-response curve provides a powerful, unified explanation for the two distinct piezoresistive regimes observed in experiments.

The "Contact Effect" (High Strain):

The stable, continuous decrease in resistance observed at moderate to high strains (e.g., 10%-30% strain) on our average-response curve is a direct computational validation of the literature's "contact effect".2 This behavior is the direct result of the progressive collapse of pores and the gradual increase in total contact area (a feature explicitly designed into our regularized RVE 1). As compression increases, more and more RVEs in the ensemble are forced into contact, adding their conductive pathways to the network and steadily lowering the total resistance.

The "Crack Effect" (Low Strain) — A Statistical-Structural Insight:

The most important finding of this work is a new, structure-based explanation for the "crack effect."

The Premise: Our model features a random distribution of protrusion lengths. By statistical necessity, a small fraction of the 200+ RVEs will be generated with very long protrusions or very small initial gaps.

The Implication: These "statistically weakest" pores will be the very first to make contact, at an extremely small compressive strain.

The Result: Before contact, the resistance of that RVE is infinite (open circuit). The instant contact is made (enabled by our abstracted rectangular protrusion), its resistance drops to a finite value. For this single RVE, the relative change, ΔR⁄R_0, is mathematically massive.

The Conclusion: The "ultra-high" sensitivity (GF > 25) reported in the literature is, therefore, not a separate physical mechanism (like coating cracks), but rather the emergent, statistical signature of the initial contact events of the most vulnerable 1-5% of the pore distribution. This is the computational analogue of the "fractured microstructure" or "tunneling effect", explained purely through stochastic geometry [1,7].

As strain increases beyond this initial regime ($\varepsilon > 1\%$), more "average" pores begin to collapse. The "contact effect" becomes dominant. The newly added conductive pathways from these later-collapsing pores represent a smaller relative change to the already-established conductive network, causing the overall GF to drop and stabilize.

Therefore, this model is the first to demonstrate computationally that the "crack effect" and "contact effect" are not two different mechanisms, but simply two sequential phases of a single statistical-structural evolution driven by the random geometry of the foam.

5 CONCLUSION AND FUTURE OUTLOOK

5.1 Summary of Contributions

We have developed and validated a new computational framework for piezoresistive foams that successfully overcomes the three primary limitations of previous models. Our model is:

Structure-Based: It explicitly models pore collapse via a novel, computationally-stable stochastic RVE.

Physically-Complete: It incorporates the essential, time-dependent mechanics of the polymer by using a Generalized Maxwell viscoelastic model fit to experimental data.

Computationally-Innovative: It employs a "segmented quasi-static" (SQS) method to solve the coupled dynamic-contact-conduction problem within standard FEM software.

The key finding of this work is that the macroscopic, predictable sensor response emerges from the ensemble average of a stochastic micro-architecture. This framework provides a unified, structural origin for the "crack" and "contact" regimes, re-interpreting them as the statistical-temporal evolution of pore collapse across a random distribution.

5.2 Limitations and Future Work

Extension to 3D: The current model is 2D, a trade-off made to achieve a large statistical sample size. The immediate next step is to extend this framework to a full 3D stochastic RVE, which will provide greater quantitative accuracy.

Incorporating Tunneling Resistance: The current model uses a binary "contact/no-contact" model, regularized by the rectangular protrusion. A more refined model would replace this binary contact with a continuous tunneling resistance model [7], where resistance is an exponential function of the gap distance. This would allow for a more precise quantitative prediction of the high-GF "crack effect" regime.

The Path to Inverse Design: This framework is currently a forward model (input: structure → output: response). Its ultimate potential, aligning with our initial goal to "infer the influence of the manufacturing process", is to be used for inverse design. By building a surrogate model trained on thousands of SQS simulations, we can begin to answer the question: "What statistical distribution of pore geometries—controlled by what manufacturing parameters—is required to produce a sensor with a specific, desired (e.g., perfectly linear) piezoresistive response?"

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

REFERENCES

[1] Niu Z, Pinfield V J, Wu B, et al. Towards the digitalisation of porous energy materials: evolution of digital approaches for microstructural design. Energy & environmental science, 2021, 14(5): 2549-2576.

[2] Cao C, Boutilier M S H, Kim S, et al. Low-profile, large-range compressive strain sensing using micromanufactured CNT micropillar arrays. ACS Applied Materials & Interfaces, 2023, 15(32): 38665-38673.

[3] Shen, Mingjie, Qin Xianrong. Research on Dynamic Compression Constitutive Model of Rigid Polyurethane Foam and Finite Element Simulation. Mechanical & Electrical Technology, 2024(1): 36-41.

[4] Yao H B, Ge J, Wang C F, et al. A flexible and highly pressure - sensitive graphene - polyurethane sponge based on fractured microstructure design. Advanced Materials, 2013, 25(46): 6692-6698.

[5] Zhang J, Wang Z, Shang C, et al. Piezoresistive relaxation and creep model of porous polymer nanocomposite supported by experimental data. Sensors and Actuators A: Physical, 2024, 366: 115002.

[6] Yan Xiaoxin. Research on mechanical and electrical properties of conductive polymer composites based on finite element simulation. Ningbo University, 2020.

[7] Tai G, Wei D, Su M, et al. Force-sensitive interface engineering in flexible pressure sensors: A review. Sensors, 2022, 22(7): 2652.

[8] Lu G, Renken L, Li T, et al. Experimental study on the polyurethane-bound pervious mixtures in the application of permeable pavements. Construction and Building Materials, 2019, 202: 838-850.

[9] Bao W S, Meguid S A, Zhu Z H, et al. Modeling electrical conductivities of nanocomposites with aligned carbon nanotubes. Nanotechnology, 2011, 22(48): 485704.

[10] Li Z, Ye J, Liu L, et al. A new three-dimensional microscopic modeling strategy in studying coupled electro-mechanical properties for piezoelectric composites. Mechanics of Advanced Materials and Structures, 2025: 1-30.