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# THE DYNAMIC MECHANISM OF HEAVE IN OSCILLATING BUOY-TYPE WAVE ENERGY DEVICES AND THE OPTIMIZATION OF ENERGY HARVESTING SYSTEMS

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**Abstract:** This paper presents a systematic study on dynamic modeling and parameter optimization to address the energy capture efficiency of wave energy conversion devices in the vertical oscillation mode. Based on Newton's second law, the study establishes a coupled second-order nonlinear differential equation system describing the motion of the float and the embedded oscillator, comprehensively considering the interactions among wave excitation forces, wave-induced damping forces, hydrostatic restoring forces, and the energy output system. To solve this strongly coupled system of equations, the fourth-order Runge-Kutta method was employed to calculate time-domain responses, yielding precise dynamic characteristics of system displacement and velocity at extremely small time steps. For evaluating energy conversion efficiency, a single-objective optimization model was formulated to maximize steady-state average output power, utilizing Riemann integration and approximation methods to handle the discretized integration of instantaneous power. By introducing a Bayesian optimization algorithm, the study achieved efficient heuristic search within the complex damping parameter space. Experimental results show that under two typical operating conditions—constant damping and power-law damping—the system achieved the expected power capture targets through optimal damping coefficient configuration, with a maximum average power of 228W. These findings provide quantitative evidence for the structural optimization of wave energy devices and the formulation of PTO system strategies.
**Keywords:** Wave energy conversion; Pitching motion response; Bayesian optimization algorithm

## 1 INTRODUCTION

As a widely distributed, abundant, and high-value clean energy source, ocean wave energy is a key component in achieving future energy transition [1,2]. However, how to efficiently and reliably convert unstable wave energy into electrical energy remains a core challenge in current ocean energy development. The primary objective of wave energy conversion devices is to capture and maximize output power through energy harvesting systems in complex and variable ocean wave environments. Although previous studies have preliminarily explored the dynamic response of such devices, they still lack a systematic and efficient mathematical framework for solving strongly coupled differential equations involving nonlinear damping terms and for global optimization across large parameter spaces. The innovation of this section lies in proposing an analytical paradigm that integrates high-precision numerical simulation with heuristic global optimization. By employing a fourth-order Runge-Kutta algorithm, we overcome the analytical challenges posed by nonlinear dynamic equations, and by combining this with Bayesian algorithms, we effectively solve the problem of optimal damping matching under costly evaluation functions [3]. The general research approach in this section is as follows: First, based on the principles of fluid mechanics and Newtonian mechanics, a mathematical description of the vertical oscillatory motion of the float and oscillator is established and transformed into a system of first-order ordinary differential equations; second, the RK4 algorithm is used to solve the time-domain motion response for the first 40 cycles at a fixed time step, and the initial equilibrium state is verified; Subsequently, an average power integration model for the steady-state interval is constructed based on the definition of instantaneous power, and a discretization approximation is applied; finally, a Bayesian optimization algorithm is employed to perform an iterative search within the feasible domain of the given damping parameters, aiming to identify the optimal design scheme that maximizes the system's average output power [4,5].

## 2 ESTABLISHMENT AND SOLUTION OF THE MODEL

### 2.1 Establishment of the Heave Motion Model

According to the problem, only the heave motion of the floater is considered. Assume that the motion directions of both the floater and the oscillator are upward, and establish the motion equations of the floater and the oscillator [6,7].
Force analysis diagram of heave motion of the oscillator and the floater is shown in Figure 1.

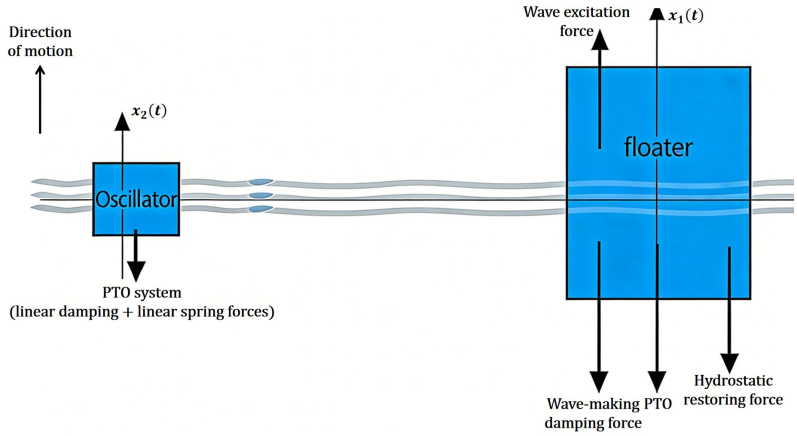


**Figure 1** Force Analysis Diagram of Heave Motion of the Oscillator and the Floater

The floater is subjected to wave excitation force $F_1$, wave-making damping force $F_2$, hydrostatic restoring force $F_3$, PTO system force $F_{PTO}$, and added inertia force $F_6$. The oscillator is subjected to linear spring force $F_4$ and linear damping force $F_5$. Based on the given information, the following equations are obtained:
The wave excitation force is expressed as:

$$F_1=f\cos\omega t \tag{1}$$

The wave-making damping force is expressed as:

$$F_2=-k_1\frac{dx_1(t)}{dt} \tag{2}$$

From the appendix provided in the problem, the hydrostatic restoring force is caused by the change in buoyancy acting on the floating body during heave motion, so it can be expressed as:

$$F_3=-\rho g\pi r^2 x_1(t) \tag{3}$$

where $\rho$ is the density of seawater, and $r$ is the bottom radius of the floater [8,9].
The linear spring force is expressed as:

$$F_4=-k_3(x_2(t)-x_1(t)) \tag{4}$$

The linear damping force is expressed as:

$$F_5=-k_4|v_2(t)-v_1(t)|^{0.5} \tag{5}$$

The PTO system force is expressed as:

$$F_{PTO}=F_4+F_5 \tag{6}$$

The added inertia force is expressed as:

$$F_6=-m_a\frac{d^2x_1(t)}{dt^2} \tag{7}$$

According to Newton's second law, the heave motion model of the floater and the oscillator can be obtained:

$$\begin{cases}(m_1+m_a)\dfrac{d^2x_1(t)}{dt^2}=F_1+F_2+F_3-F_{PTO}\\ m_2\dfrac{d^2x_2(t)}{dt^2}=F_{PTO}\end{cases} \tag{8}$$

Convert Equation (8) into the standard form of a first-order ordinary differential equation system:

$$\begin{cases}\dfrac{dx_1(t)}{dt}=v_1(t)\\ \dfrac{dx_2(t)}{dt}=v_2(t)\\ \dfrac{dv_1(t)}{dt}=\dfrac{1}{m_1+m_a}\left(f\cos\omega t-k_1v_1(t)-\rho g\pi r^2x_1(t)-k_3(x_2(t)-x_1(t))-k_4|v_2(t)-v_1(t)|^{0.5}\right)\\ \dfrac{dv_2(t)}{dt}=\dfrac{1}{m_2}[k_3(x_2(t)-x_1(t))+k_4|v_2(t)-v_1(t)|^{0.5}]\end{cases} \tag{9}$$

### 2.2 Solution of the Heave Motion Model

It is easy to find that this ordinary differential equation system involves nonlinear terms and has a high degree of coupling, making it difficult to obtain an analytical solution. Therefore, the numerical solution method is adopted here. To ensure accuracy, the fourth-order ODE solution method, namely the fourth-order Runge-Kutta method (RK4), is selected [10].
The standard form of the fourth-order Runge-Kutta method is as follows:

$$\begin{cases} t_{n+1}=t_n+h \\ k_1=f(y_n,t_n) \\ k_2=f\left(y_n+\frac{h}{2}k_1,t_n+\frac{h}{2}\right) \\ k_3=f\left(y_n+\frac{h}{2}k_2,t_n+\frac{h}{2}\right) \\ k_4=f(y_n+hk_3,t_n+h) \\ y_{n+1}=y_n+\frac{h}{6}(k_1+2k_2+2k_3+k_4) \end{cases} \tag{10}$$

The initial conditions are:

$$\begin{cases} x_1(0)=0 \\ x_2(0)=0 \\ \left.\frac{dx_1(t)}{dt}\right|_{t=0}=0 \\ \left.\frac{dx_2(t)}{dt}\right|_{t=0}=0 \end{cases} \tag{11}$$

#### *2.2.1 Solution to dynamic modeling and motion simulation of heave motion for float-oscillator system with constant damping*

The incident wave frequency $\omega=1.4005\ s^{-1}$, and the damping coefficient of the linear damper $k_4=10000\ N\cdot s/m$. Substitute the parameters into Equation (9) and solve it via RK4 to obtain the numerical solutions for the first 40 wave periods. Results is shown in Table 1.

**Table 1** Results of Dynamic Modeling and Motion Simulation of Heave Motion for Float-Oscillator System with Constant Damping

| Time | Floater displacement | Floater velocity | Oscillator displacement | Oscillator velocity | Relative displacement | Relative velocity |
|---|---|---|---|---|---|---|
| 10 | -0.19071 | -0.64101 | -0.21168 | -0.69395 | -0.02097 | -0.05295 |
| 20 | -0.59068 | -0.24095 | -0.63425 | -0.27278 | -0.04356 | -0.03182 |
| 40 | 0.28537 | 0.31297 | 0.29650 | 0.33291 | 0.01113 | 0.01994 |
| 60 | -0.31451 | -0.47946 | -0.33144 | -0.51573 | -0.01693 | -0.03627 |
| 100 | -0.08362 | -0.60421 | -0.08407 | -0.64300 | -0.00045 | -0.03879 |

Relationship between heave displacement and velocity under constant damping coefficient is shown in Figure 2.

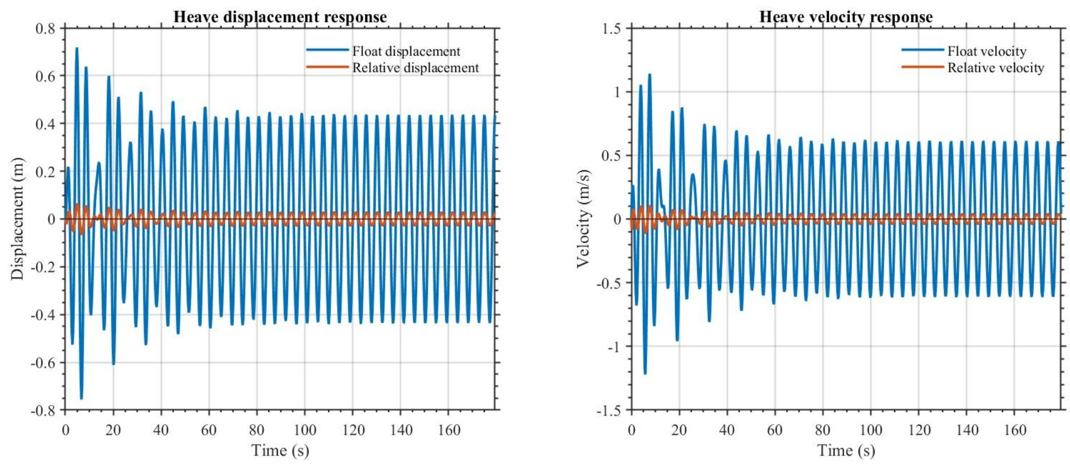


**Figure 2** Relationship between Heave Displacement and Velocity under Constant Damping Coefficient

#### *2.2.2 Solution to dynamic modeling and motion simulation of heave motion for float-oscillator system with power-law damping*

Substitute the linear damper damping coefficient $k_4=10000|v_2(t)-v_1(t)|^{0.5}$ into Equation (9) with other parameters unchanged, and solve to obtain the numerical solutions for the first 40 wave periods.

Results of Dynamic Modeling and Motion Simulation of Heave Motion for Float-Oscillator System with Power-Law Damping is shown in Table 2.

**Table 2** Results of Dynamic Modeling and Motion Simulation of Heave Motion for Float-Oscillator System with Power-Law Damping

| Time (s) | Floater displacement (m) | Floater velocity (m/s) | Oscillator displacement (m) | Oscillator velocity (m/s) | Relative displacement (m) | Relative velocity (m/s) |
|---|---|---|---|---|---|---|
| 10 | -0.20588 | -0.65282 | -0.23457 | -0.69994 | -0.02870 | -0.04712 |
| 20 | -0.61111 | -0.25478 | -0.66106 | -0.27702 | -0.04995 | -0.02224 |
| 40 | 0.26877 | 0.29530 | 0.28016 | 0.31252 | 0.01139 | 0.01722 |
| 60 | -0.32716 | -0.49152 | -0.34961 | -0.52559 | -0.02244 | -0.03407 |
| 100 | -0.08841 | -0.60983 | -0.09349 | -0.65008 | -0.00508 | -0.04025 |

Relationship between heave displacement and velocity under proportional damping coefficient is shown in Figure 3.

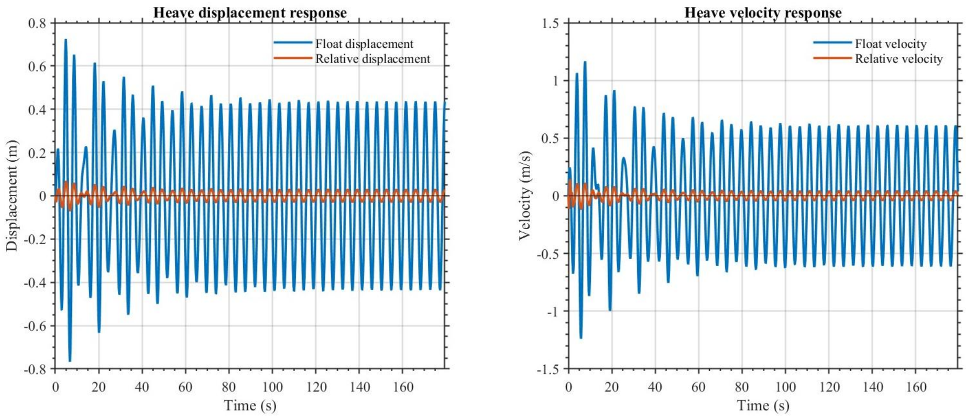


**Figure 3** Relationship between Heave Displacement and Velocity under Proportional Damping Coefficient

## 3 OPTIMIZATION OF DAMPING COEFFICIENTS FOR MAXIMUM AVERAGE OUTPUT POWER IN HEAVE MOTION

### 3.1 Establishment of the Model

#### *3.1.1 Calculation of average power*

The instantaneous power is the product of the damping force and the relative velocity:

$$P_P(t)=F_5V \tag{12}$$

Take $t_0>t_w$ (a moment after the system reaches steady-state vibration), and calculate the average power over a period starting from $t_0$:

$$\overline{P_P}=\frac{1}{T}\int_{t_0}^{t_0+T} C|v_r(t)|^{(2+\alpha)}dt \tag{13}$$

where C∈[1,100000], α∈[0,1].

#### *3.1.2 Establishment of the damping coefficient optimization model*

Take the average output power as the objective function to maximize the average output power of the PTO system:

$$\overline{P_P}=\max\frac{1}{T}\int_{t_0}^{t_0+T} C|v_2(t,C)-v_1(t,C)|^{2+\alpha}dt \tag{14}$$

Subject to constraints:

$$\begin{cases}(m_1+m_a)\dfrac{d^2x_1(t)}{dt^2}=F_1+F_2+F_3-F_{PTO}\\ m_2\dfrac{d^2x_2(t)}{dt^2}=F_{PTO}\\ F_1=f\cos\omega t\\ F_2=-k_1\dfrac{dx_1(t)}{dt}\\ F_3=-\rho g\pi r^2x_1(t)\\ F_{PTO}=F_4+F_5\\ F_4=-k_3(x_2(t)-x_1(t))\\ F_5=-k_4|v_2(t)-v_1(t)|^{0.5}\end{cases} \tag{15}$$

### 3.2 Solution of the Model

The integral is discretized using the Riemann sum approximation:

$$\overline{P_P}=\max\frac{1}{T}C\sum_{i=t_0}^{t_0+T}|v_2(i,C)-v_1(i,C)|^{2+\alpha}h \tag{16}$$

The Bayesian optimization algorithm is used for iterative search to find the optimal parameters.
Flowchart of Bayesian optimization algorithm is shown in Figure 4.

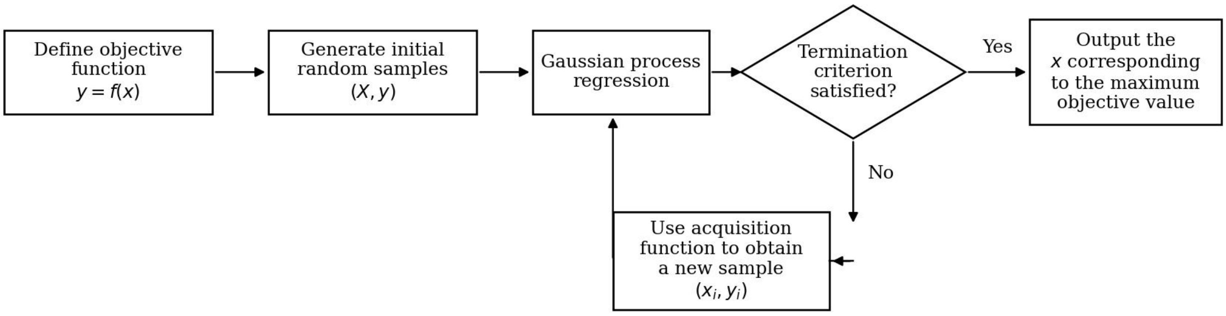


**Figure 4** Bayesian Optimization Algorithm

***3.2.1 Optimization of constant damping coefficient for maximum output power***

When α=0:

$$\overline{P_P}=\max\frac{1}{T}C\sum_{i=t_0}^{t_0+T}|v_2(i,C)-v_1(i,C)|^{2}h \tag{17}$$

The result shows that when the damping coefficient is 36887.4, the maximum output power is 228.31 W. Functional relationship between power and damping coefficient is shown in Figure 5.

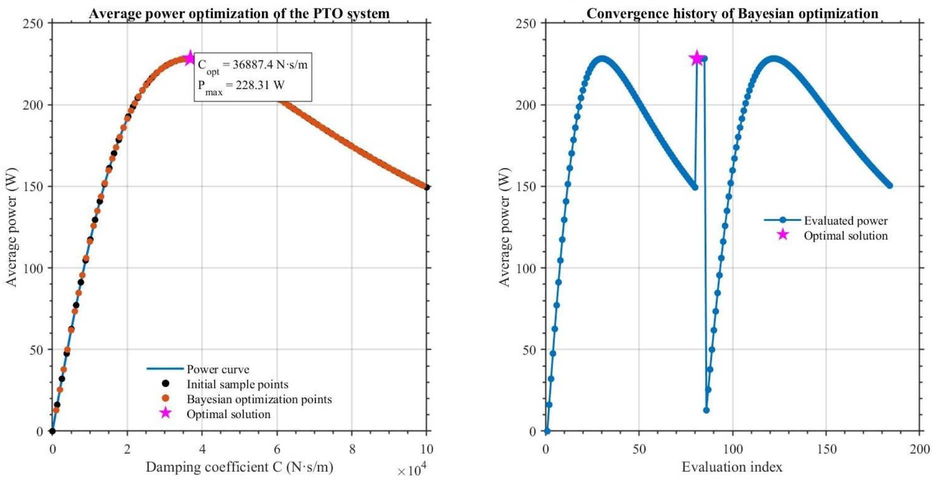


**Figure 5** Functional Relationship between Power and Damping Coefficient

***3.2.2 Optimization of power-law damping coefficient for maximum output power***

When α∈[0,1]:

$$\overline{P_P}=\max\frac{1}{T}C\sum_{i=t_0}^{t_0+T}|v_2(i,C)-v_1(i,C)|^{2+\alpha}h \tag{18}$$

The result shows that when the damping coefficient is 36865.7 and α=0.0002, the maximum output power is 228.22 W. Optimization process is shown in Figure 6.

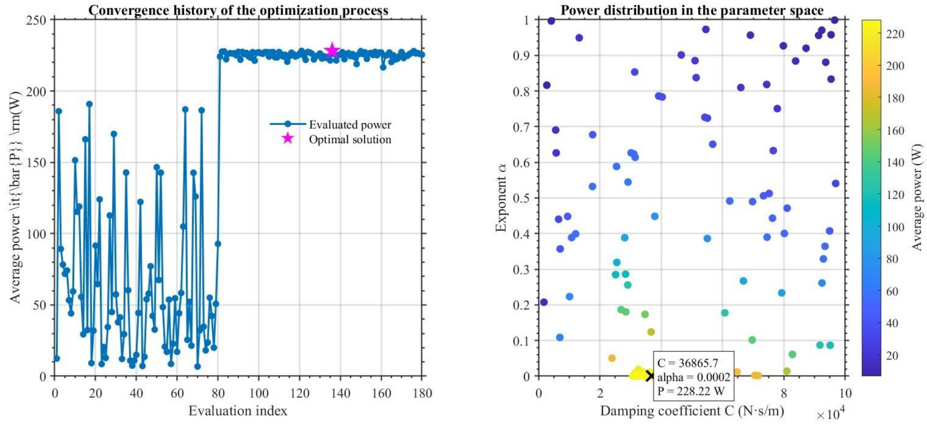


**Figure 6** Optimization Process

Bivariate functional relationship between power, damping coefficient and $\alpha$ is shown in Figure 7.

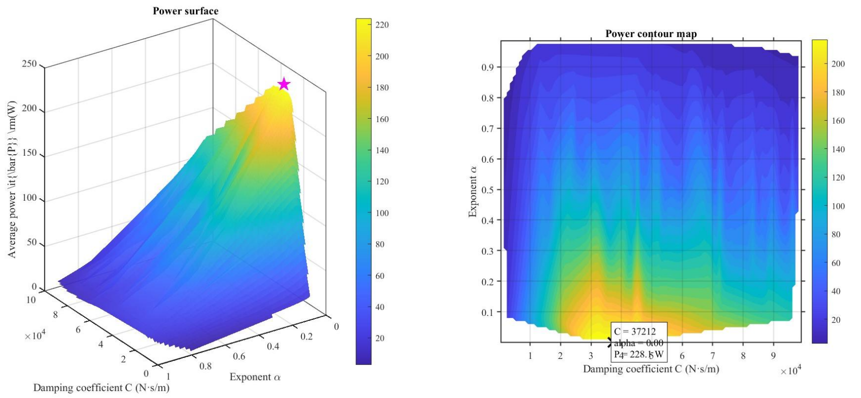


**Figure 7** Bivariate Functional Relationship between Power, Damping Coefficient and $\alpha$

## 4 CONCLUSIONS

This study systematically completed the response analysis and power optimization of wave energy devices under the heave degree of freedom by constructing a rigorous heave dynamics model and a Bayesian optimization framework. The research confirms that the RK4 numerical algorithm can accurately characterize the time-domain characteristics of nonlinear systems, while the Bayesian optimization algorithm possesses significant efficiency advantages over traditional algorithms in handling strongly coupled systems and evaluation functions with high computational costs. However, this study still has certain limitations: the model currently assumes seawater to be an ideal, non-viscous fluid, neglecting the damping effect of viscous resistance on system amplitude in actual ocean conditions; furthermore, the current wave excitation model is primarily based on single-frequency linear regular waves and has not fully considered the stochastic characteristics of the wave spectrum in real sea conditions. Future research should focus on incorporating dynamic meteorological coupling factors and irregular wave dynamics criteria, and further explore the energy transfer mechanisms under multi-degree-of-freedom coupling of heave and pitch motions, with the aim of developing an intelligent wave energy harvesting decision-making system capable of all-weather adaptability.

## COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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